A cognitive analysis of dragging practises in Cabri environments

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Abstract:
Dragging in Dynamical Geometry Software (DGS) is described by introducing a hierarchy of its functions. This is suitable for classifying different attitudes and aims of students who investigate a geometric problem, such as exploring, conjecturing, validating and justifying. Moreover the hierarchy has cognitive features and can be used to describe the twofold modalities, namely ascending and descending in which students interact with external representations (e.g. Cabri drawings). Switching from one modality to the other through dragging often allows them to produce fruitful conjectures and to pass from the empirical to the theoretical side of the question. The genesis of such different functions in students does not happen automatically but is the consequence of specific didactical interventions of the teacher in the pupils' apprenticeship of Cabri practises. A worked-out example illustrates the theoretical concepts introduced in the paper.

Kurzreferat:
Der Text führt eine Funktionshierarchie für den Gebrauch des Zugmodus in Dynamischer Geometrie Software (DGS) ein. Diese ist zur Klassifikation von Verhaltensweisen und Zielen der Lernenden geeignet, wenn diese geometrische Problem explorieren, Vermutungen aufstellen, bestätigen und rechtfertigen und dabei theoretische und empirische Aussagen in Cabri-Zeichnungen in Beziehung setzen (wechselweise „auf-" und „absteigen"). Der Wechsel zwischen diesen Sichtweisen unter Nutzung des Zugmodus erlaubt es ihnen oft, gehaltvolle Vermutungen zu formulieren und von einer empirischen zu einer theoretischen Sicht auf das Problem zu wechseln. Der Sichtwechsel stellt sich allerdings nicht automatisch ein, sondern ist die Folge besonderer didaktischer Maßnahmen des Lehrenden. Ein ausgearbeitetes Beispiel illustriert die neu eingeführten theoretischen Begriffe.

Introduction

We sketch the different functions of dragging in Cabri environments and introduce a hierarchy suitable for classifying them and for describing some of their cognitive features in learning processes. Dragging reveals crucial in the dialectic of perceptual vs. theoretical aspects which feature the whole geometrical reasoning. In fact, as C. Laborde underlines (Laborde, 1999), diagrams in geometry play an ambiguous role: one the one hand, they refer to theoretical objects, whereas on the other hand they offer *graphical - spatial* properties which can give rise to a perceptual activity from the individual.

Within dynamic geometry software, the interaction concerns deeply perceptual aspects, which involve not only the objects (e. g. drawings) but also the physical perceptions of students, their motions, gestures, languages… and the artefacts that they use as mediating instruments (fig.1).

The nature of the relationship between the perceptive and the theoretical level is complex and requires a fresh analysis, which entails different components: didactical, cognitive, epistemological. Perceptual aspects which must be analysed concern many components, i.e. visual phenomena, motion, kinaesthesia, inner time(s); on the other side, the most typical theoretical features are the structured mathematical objects, their invariant properties, conjectures, theorems, proofs. We will concentrate here only in the role played by dragging; for a survey of the other aspects, see Arzarello (2001).
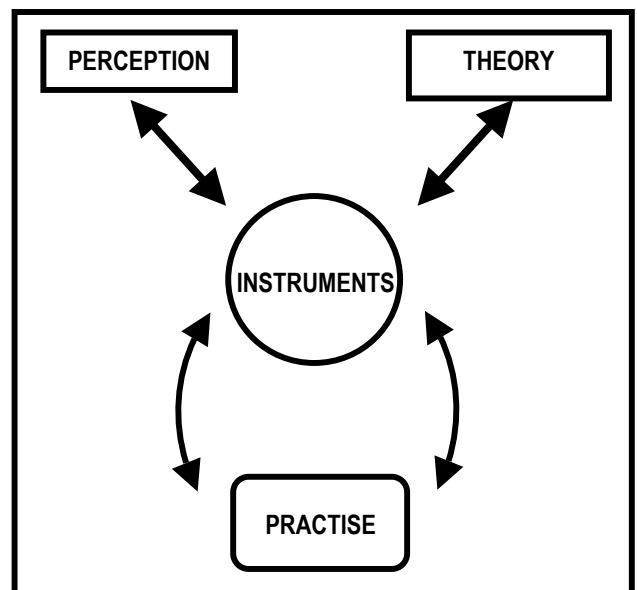

Fig.1

1. Dragging practises from a cognitive standpoint

Let us enter specifically into dragging activities: the main points of the analysis can be so summarised:

such practises as dragging can mediate the relationship theoretical-perceptual in a specific way, creating entities with a new status (see the Structured Mathematical Objects, defined in Arzarello, 2000);

these emerge from problem solving and have institutional and personal features (Mariotti, 2001).

In fact, dragging supports the production of conjectures: exploring drawings by moving them, looking at the ways after which their forms change (or do not change), allows users to discover their invariant properties. The possibility of dragging offers a feedback to the discovering phase, and in this way it provides support to the role of proofs as real "explanations" of conjectures or properties.

These computer-supported practises can be framed within a cognitive evolution back and forth from perceptions to abstract ideas; in fact, there are two main cognitive typologies, which can be differently faded according to the concrete situation (Saada-Robert, 1989;

Arzarello, 2000; Olivero, 1999):

ascending processes, from drawings to theory, in order to explore freely a situation, looking for regularities, invariants, etc.

descending processes from theory to drawings, in order to validate or refute conjectures, to check properties, etc.

Ascending and descending processes shown by dragging practises in Cabri reveal cognitive shifts from the perceptual level to the theoretical one and back in students' mathematical activity. Ascending and descending modalities vary during the performance and mark also the way subjects look at what is considered as given and at what is supposed to be found. They constitute a delicate cognitive point, which has also a relevant didactic aspect. It is precisely in these two aspects that one can observe different dynamics between 'pencil & paper' and 'Cabri' environments. In both, the transition is ruled by *abduction*[1]; but while in the former the abductions are produced because of the ingenuity of the subjects, in Cabri the dragging process can mediate them (see Arzarello et al., 1998). Moreover, such a repeated switching supports the evolution from perceptions towards a more theoretical frame: this evolution is marked by a kind of rhythm from ascending to descending modalities and back (Arzarello, 2000).

2. A hierarchy of the dragging modalities

In previous researches (Arzarello et al, 1998b; Olivero, 1999; Arzarello, 2001), developing Hölzl research (1995, 1996), we have identified different types of dragging which students use according to their different purposes during the solution process of open problems. More specifically, observing how students use the mouse while solving a problem in Cabri, we have found the following modalities (see § 3 for an example):

- *Wandering dragging*: moving the basic points on the screen randomly, without a plan, in order to discover interesting configurations or regularities in the drawings.

Bound dragging: moving a semi-dragable[2] point (it is already linked to an object).

Guided dragging: dragging the basic points of a drawing in order to give it a particular shape.

Dummy locus dragging: moving a basic point so that the drawing *keeps* a discovered property; the point which is moved follows a path, even if the users do not realise this: the locus is not visible and does not 'speak' to the students, who do not always realise that they are dragging along a locus.

Line dragging: drawing new points along a line in order to keep the regularity of the figure.

Linked dragging: linking a point to an object and moving it onto that object.

Dragging test: moving dragable or semi-dragable points in order to see whether the drawing keeps the initial properties. If so, then the figure passes the test; if not, the drawing was not constructed according to the geometric properties you wanted it to have.

We have observed that students exploited these different dragging modalities in order to achieve different aims, such as exploring, conjecturing, validating, justifying. For example, *wandering* and *guided dragging* were generally used in the discovering phase, *dummy locus dragging* marked the construction of a conjecture, *dragging test* was mainly used to test a conjecture. Therefore looking at how students used dragging provided an insight into their cognitive processes.

To give an idea of what we mean let us illustrate these dragging modalities through a nice example, due to Hölzl (1995; 1996); see also Olivero (1999).

TASK: *You are given a triangle ABC. Consider a point P on AB and the two triangles APC and PCB. Make an hypothesis about the properties of ABC which are necessary so that both APC and PCB are isosceles (such triangles are called 'separable').*

This open problem of exploration may lead the solver to a range of discoveries, where one can see the different modalities of dragging.
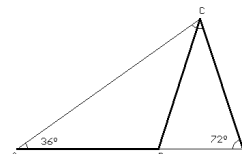
First, one can find two different configurations:

fig.2: AP=PC=CB   fig.3: AP=PC=PB
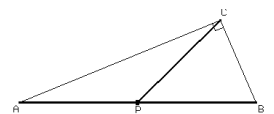
Let us investigate the configuration of fig.3. What are the characteristics of the triangle in fig.3?

In Cabri you draw a triangle ABC and P as the midpoint of AB. Then you start moving the point C, which is a 'dragable point' (Hölzl, 1996), all around the screen in order to see whether such triangles (the 'separable' ones) exist. You are doing **wandering dragging.** Through this way of dragging, you find many triangles that satisfy the property (fig. 4), so now you are sure that the task has a solution.

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[1] The following example (Peirce, 1960, p.372) is illuminating about abduction. Suppose I know that a certain bag is plenty of white beans. Consider the sentences: A) these beans are white; B) the beans of that bag are white; C) these beans are from that bag. A deduction is a concatenation of the form: B and C, hence A; an abduction is: A and B, hence C (Peirce called hypothesis the abduction). An induction is: A and C, hence B. For more details, see Magnani et al. (2001) and Arzarello et al. (2000).

[2] A semi-dragable point is a point linked to an object, that can be moved but only on the object it belongs to.
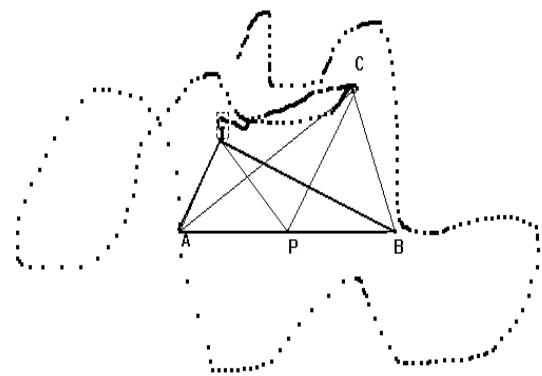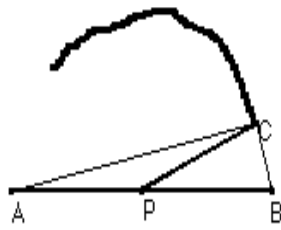
Fig. 4

Fig. 5

Now you go on moving C and you stop when the triangle ABC is 'separable', such as when PC equals AP. Looking at the drawing you start thinking about the characteristics of this triangle. In order to know more, you try to drag C in such a way that ABC keeps its property (PC=AP). In other words, your dragging is no more 'by chance': you feel that you are moving so that the triangle ABC continues to be divided into two isosceles triangles. Therefore you are using **dummy locus dragging**: the path followed by C cannot be drawn and at the moment it is dummy. To understand the situation better you can exploit the **line dragging** by marking the points correspondent to the positions occupied by C when ABC has the right property (fig. 6). Now you observe that they seem to lie on a circle, exactly on the circle centred in P with radius PA (=PB). The *dummy locus* now becomes explicit; by constructing the circle with centre P and radius PA you see that the points previously marked really are on that circle (fig. 7).
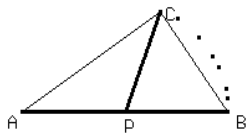

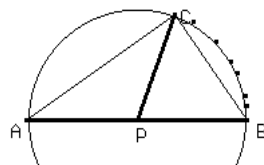Fig. 6   Fig. 7

So you conjecture that the 'separable' triangles are those whose vertex C belongs to a circle with centre P and with radius PA. This condition is equivalent to the fact that ABC is inscribed in a circle with AB as a diameter, which means that the angle <ACB is right. Therefore your conjecture can now be more precisely formulated in a logical form: if ABC is right-angled (<ACB=90°) then it is 'separable'.

In Cabri you have got a way to validate this statement. By linking the vertex C to the circle discovered by means of line dragging and moving it on that, so using **linked dragging**, you can see that all the triangles that are continuously redrawn satisfy the property of being 'separable' (fig. 8).

As a last step you can construct a right-angled triangle ABC, the middle point P of AB, the segment PC and use the **dragging test**, such as move the triangle through all its dragable points and observe that it keeps the asked property (fig.9).

A first observation can be drawn at this level of analysis, based thoroughly on perceptive facts: there is a 'genetic' hierarchy in the use of these dragging modalities, in that the solution process develops through a sequence of different modalities. Of course, this 'genesis' is not pre-scriptive, in that not all problem solvers will undertake it. The evolution from perceptive to theoretical aspects through ascending/descending modalities does not happen automatically, but a careful didactical design is needed to support students in such a process within dynamic geometry environments (see Mariotti & Bartolini, 1998).

A second observation is that dragging in Cabri seems to show at a perceptive level what the students' cognitive processes are. Actually, we realised that they exploit these different dragging modalities in order to achieve different aims.

Wandering, bound and guided dragging are used to investigate and explore a given task, so they are part of the ascending control stream.

Dummy locus dragging can be seen as a wandering dragging which has found its path; the trace of this dragging represents, at an empirical level, a dummy locus that is not yet visible to the subject. A *dummy locus* can
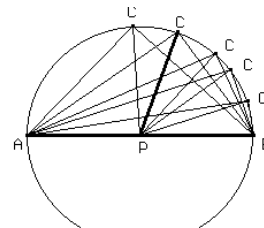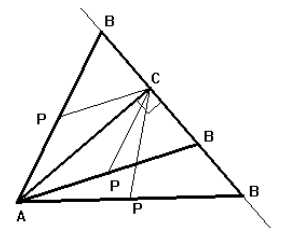

Fig. 8   Fig. 9

act both as a producer of new powerful heuristics (Holzl, 1996) and as a logical reorganiser of the previous investigations (Pea, 1987). This modality reveals the beginning of the shift from ascending control to descending control, that is an abduction. The solver is beginning to see a certain relation/property/invariance and he is trying to make sense of it in logical terms. If drawn figures keep some regularity, then the point C describes a certain locus L. If C runs on L, then the corresponding drawings F(C) show some regularity, invariance or rule. Therefore *dummy dragging* supports students in producing abductions, and as a consequence in the transition between the two modalities of control.

Line dragging follows dummy locus dragging, as it makes the locus explicit and visible on the screen; it is part of the process of transition towards descending control.

Linked dragging allows the subject to check his conjecture: if the locus can be constructed in Cabri (e.g. a line, a circle), the subject can link the point to that locus, and the discovered property must be kept through dragging it on the locus. Therefore it reveals the beginning of descending control. On the other hand it can also be exploited in the exploration process, for example if you want to decrease the free parameters of the initial situation.

Dragging test is used as a means of validating a conjecture, in particular conjectures that are originated by a visual or a construction, therefore it reveals a kind of descending control.

At the end of this analysis, it is clear that the transition from one dragging modality to another shows a 'genesis' which is connected with the cognitive ascending / descending modalities described above: for example wandering dragging is typical of an ascending modality, while a test dragging is typical of a descending modality (see fig. 10 on the page before).
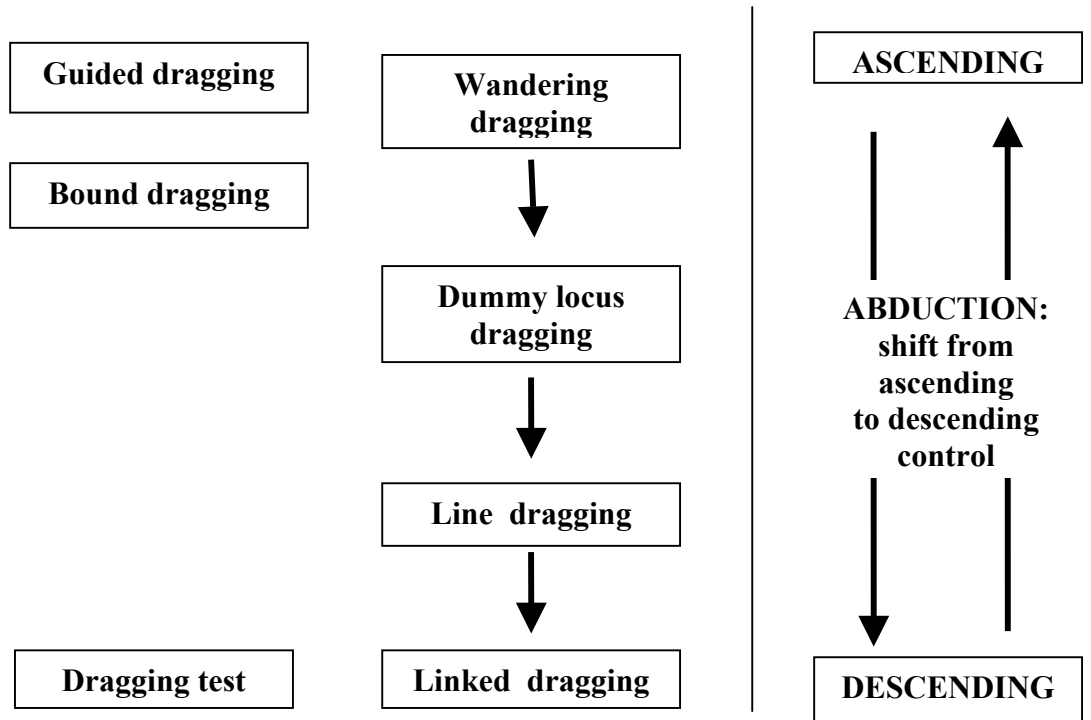
Fig. 10

3. A worked-out example

We shall now illustrate with one examples (taken from Olivero, 1999) how such a hierarchy can be used in everyday classroom activity to read what happens when students are using Cabri to explore open situations.

TASK: Let ABCD be a quadrilateral. Consider the bisectors of its internal angles and their intersection points H, K, L, M of pairwise consecutive bisectors. Drag ABCD, considering all its different configurations: what happens to the quadrilateral HKLM? What kind of figure does it become?

This problem was given to a class of 27 students (15 years old), who were asked to solve it working in pairs at the computer. After one hour a classroom discussion took place in order to discuss the conjectures and proofs produced by the students. An observer (the author) took notes of the activity of two pairs of students. We analyse the solving process of one pair of students. For a "transcript" of the process see "attachment" on top of the next page.

The process that takes place can be summarised as follows:

a) First, they see (in Cabri) that:

If H, K, L ,M are coincident (one point) then the sum of two opposite sides equals the sum of the other two.

b) Then they make an ABDUCTION, that is 'quadrilaterals that can be circumscribed to a circle'.

c) Finally, they produce a conjecture in a logical form:

If ABCD circumscribed then H, K, L, M coincide (one point).

Such an analysis is a powerful tool to investigate the cognitive processes of pupils through visible actions. In our observations we have seen a continuous shift from working in Cabri to theoretical reasoning. In some instances the students anticipate a conjecture and then they check it in Cabri, while in other cases they 'read' what is happening in Cabri and make conjectures from this. The *dummy locus dragging* proved to be quite important in the discovering process, as it is the crucial point when students are able to make a conjecture, after the locus they 'feel' while moving in Cabri according to certain properties. Students seemed to exploit this tool spontaneously, however it is not always like this. Sometimes it seems necessary the intervention of the teacher to introduce some of the dragging tools or modalities to students. In fact, these tools need to be made explicit and introduced into the classroom culture, so that they may become available to all students, it is necessary a *cultural carving* of students' perceptions through a cognitive apprenticeship and the role of the teacher in this is crucial.

Other examples are discussed in Arzarello et al. (1998), Arzarello (2000), Olivero (2001, 2002).
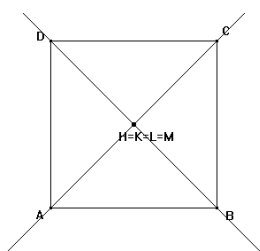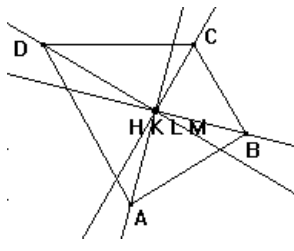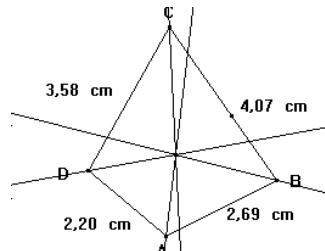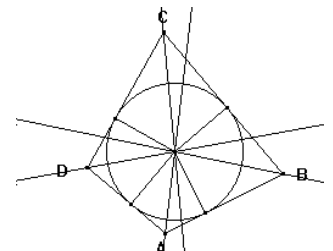

Fig. 11   Fig. 12   Fig. 13   Fig. 14

Attachment WHAT STUDENTS DID	DRAGGING and COGNITIVE MODALITIES
Episode 1	
At first students start exploring the situation by examining standard cases. It seems they are following an implicit rule: When ABCD is a parallelogram, HKML is a rectangle When ABCD is a rectangle, HKLM is a square. When ABCD is a square, HKLM is a point (fig.11).	They use **guided dragging** in order to get different shapes of ABCD. **Ascending control** is guiding their experiments, as their aim is to get some conjectures about the configuration. The last step allows them to see a degenerate case: HKLM disappears into one point.
Episode 2	
As soon as they see that HKLM becomes a point when ABCD is a square, they consider it an interesting fact, therefore they drag ABCD (from a square) so that H, K, L, M keep on being coincident. They realise that this kind of configuration can be seen also with quadrilaterals that apparently have not any common property (fig. 12).	Now a regularity is discovered; so they use *dummy locus dragging*. They drag ABCD so to keep the property they have just found out. They are still in the stream of **ascending control**, as they are exploring the situation, but now they have a plan in their mind: they look for some common properties to all those figures which make HKLM one point.
Episode 3	
Paying attention to the measures of the sides of the figure ABCD (which appear automatically next to the sides and change in real time, while dragging), they see that the sum of two opposite sides equals the sum of the other two (fig. 13); they remember that this property characterises the quadrilaterals that can be circumscribed to a circle.	Even if the locus is not explicitly recognised by the students, it is this kind of dragging that allows them to discover some regularity of the figures. Here they make an abduction, because they select 'which rule it is the case of': this is the case of circumscribed quadrilaterals. Referring to the example by Peirce, we can say that: A is "the sum of two opposite sides equals the sum of the other two", B is "a quadrilateral is circumscribed to a circle if and only if the sum of two opposite sides equals the sum of the other two", i.e. something you know while C is "these quadrilateral are circumscribed". Their reasoning is: A & B, then C. Once they have selected the right geometric property, they can 'conclude' that this is the case of circumscribed quadrilaterals. The conditional form is virtually present: its ingredients are all alive, but their relationships are still reversed, with respect to the conditional form; the direction after which the subjects see things is still in the stream of the exploration through dragging, the control of the meaning is ascending, namely they are looking at what they have explored in the previous episodes in an abductive way.
Moreover, using the Cabri menu, they construct the perpendicular lines from the point of intersection of the angle bisectors to the sides of ABCD: they see that this point has the same distance from each side of ABCD, then they draw the circle which has this length as radius: it is the circle inscribed in ABCD. After that they formulate a conjecture: *If the external quadrilateral can be circumscribed to a circle, then its internal angle bisectors will all meet in one point, so the distances from this point are equal and the sum of the opposite sides is equal too*.	The direction of control now changes: here students use the construction modality (and the consequent dragging test) to check the hypothesis formulated through abduction and at the end they write down a sentence in which the way of looking at figures has been reversed. By *dummy locus* dragging, they have seen that when the intersection points are kept to coincide the quadrilateral is always circumscribed to a circle. Now they formulate the conjecture in a logical way, which reverses the stream of thought: if the quadrilateral is circumscribed then the points coincide.
Episode 4	
At the end they construct a circle, a quadrilateral circumscribed to this circle, its angle bisectors and they observe that all of them meet in the same point (fig. 14).	At the end they check their conjecture. Now they are using the dragging test and their actions show descending control.

Concluding remarks

The first thing to be observed is the two-fold evolution of students' use of dragging in Cabri: within a session and over the whole intervention. It was observed that at the beginning the students did not use dragging in Cabri very much. This is a behaviour which has been observed in many experiments with students of different school levels but all of them just starting to use Cabri. They need to learn to move things around before they start doing this. Then an evolution of the use of dragging can usually be

seen. An hypothesis to be investigated is that students need to 'internalise' (Vygotsky, 1978) the dragging function in order to be able to use it in a productive way. That is, at first dragging could even be seen as an element which distracts and interferes, since one is not used to see objects moving on paper; then one starts experimenting with dragging but until a conscious use of it is reached, it is unlikely that students can really exploit it for what they want. Moreover, the use of different dragging modalities provides different approaches to the solution of a problem and different 'results' in terms of getting to a solution of the problem. These results suggest that these modalities might become objects of study among teachers and eventually object of teaching in the classroom, such that they become part of the classroom culture and a tool accessible to all students. The way of doing this still needs to be studied. In this respect, the role of the teacher becomes very important, as he/she is the one in charge of structuring didactical sequences such that the 'internalisation' takes place and becomes part of the classroom culture.

Our learning sequences were generally structured in way that alternated the Cabri sessions and classroom discussions or group work in the classroom. Besides, sometimes students felt the need for using other tools together with Cabri, such as sketches on paper or hand gestures (quite a lot of sketches were done on the worksheets). From these observations, it is likely that Cabri cannot provide a self-contained environment, but other tools and environments need to be used and studied in relation to Cabri. This issue has important didactical consequences, as it determines how Cabri is used in the classroom.

Our classroom experiments have shown that the software itself does not grant the transition from empirical to generic objects, from perceptive to theoretical level. On the contrary, the teacher plays a very important role in students' approach to theoretical thinking. Technology itself cannot bring about an educational change. Very often there is the belief that if the technology used is *good*, then didactics will certainly improve. This assumption does not recognise that a computer based learning environment may be very complex, may need sometime to be usefully exploited (both by the teacher and the students), according to set learning objectives. Generally speaking, using new technologies in the classroom implies the redefinition of contents, methods and of the role of the teacher (Bottino & Chiappini, 1995; Noss, 1995). Simply making a software available does not mean that people will more or less automatically take advantage of the opportunities that it affords (Perkins, 1985).

For example, dragging in Cabri allows students to validate their conjectures; therefore the function of convincing (themselves, a friend or an enemy) proof has in mathematics is no longer useful. The work in Cabri is enough for the students to be convinced of the validity of their conjectures. If the teacher does not motivate students to find out *why* a conjecture (proposition) is true, then the justifications given by students may remain at a perceptive-empirical level: the proposition is true because the property observed on the Cabri figure stays the same

when dragging the drawing, given the hypotheses do not change. When such a belief is shared in the classroom, then Cabri might become an obstacle in the transition from empirical to theoretical thinking, as it allows validating a proposition without the need to use a theory. However, if the teacher makes explicit the role of proof in justifying, then students will be motivated to prove *why* a certain proposition is true (within a theory), after they know *that* it is true (within the Cabri environment). To paraphrase Polya (1954), first we need to be convinced that a proposition is true, then we can prove it.

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