SHAPING A MULTI-DIMENSIONAL ANALYSIS OF SIGNS

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The purpose of this paper is to introduce two perspectives for observing students' activity, taking into account the main elements of their communication and thinking processes: gestures, speech, written words, mathematical signs. These perspectives give origin to two different types of analyses called parallel and serial analysis, in order to distinguish between a research attention to the contemporary production of gestures and words, or to the functions of signs, sequentially introduced during the activity.

INTRODUCTION AND BACKGROUND

Recently the analysis of gestures and their role in the construction of meanings has become relevant not only in psychology, but also in mathematics education. Gestures are considered in relation with speech, and with the whole environment where mathematical meanings grow: context, artefacts, social interaction, discussion, etc. Mathematics, as an abstract matter, has to come to terms with our need for seeing, touching, and manipulating. It requires perceivable signs and so the environment reveals crucial in the teaching-learning process.

In this paper, we elaborate on two different ways to look at the cognitive processes of students when they communicate and reason on a mathematical activity. We propose a theoretical frame shaped by the encounter of some perspectives, developed in the disciplines of mathematics education, psychology, neuroscience, and semiotics. In particular, the theoretical notions we use here are the following: from psychology, the *Information Packaging Hypothesis* (Alibali, Kita & Young, 2000); from semiotics, the idea of *semiotic means of objectification* (Radford, 2003) and that concerning the different functions of signs, i.e. *iconic, indexical* and *symbolic* (Peirce, 1955; Radford, 2003) from psycho-linguistic, the distinction between *linguistic* and *extra-linguistic* modes of expression (Bara & Tirassa, 1999). Let us sketch them to our purpose; a detailed account is given in the introduction of the present research forum.

In psychological research, Alibali, Kita & Young (2000) consider the Information Packaging Hypothesis (IPH) to describe the way gesture may be involved in the conceptual planning of the messages. According to the IPH, gesture helps speakers to "package" spatial information into verbalisable units, allowing for alternative ways of encoding and organising spatial and perceptual information. Within the similar perspective that gestures play an active role not only in speaking, but also in thinking, *gesture-speech matches* and *mismatches* are defined (Goldin-Meadow, 2003). A match occurs when all the information conveyed by a gesture is also expressed in the

uttered speech; a mismatch happens in all the other cases. Mismatches are the most interesting since they appear to be a stepping-stone on the way toward mastery of a task. But gestures are also significant from the side of semiotics if seen as *signs*. Vygotsky (1997) already pointed out that "a gesture is specifically the initial visual sign in which the future writing of the child is contained as the future oak is contained in the seed. The gesture is a writing in the air and the written sign is very frequently simply a fixed gesture" (p. 133). Nevertheless, semiotics is useful to analyse gestures only if does not forget their cultural and embodied aspects. Such a direction has been followed in mathematics education by Radford (2003) with the introduction of the so-called semiotic means of objectification. These semiotic means are constituted by different types of signs, e.g. gestures, words, drawings, and so on. They have been introduced to give an account of the way students come to generalise numeric-geometric patterns in algebra. Different kinds of generalisation have been detected. Among them, the so-called *contextual generalisation*, which still refers heavily to the subject's actions in time and space, within a precise context, even if he/she is using signs who could have a generalising meaning. In contextual generalisation, signs have a two-fold semiotic nature: they are becoming symbols but are still indexes. These terms come from Pierce (1955) and Radford (2003). An *index* gives an indication or a hint on the object: e.g. an image of the Golden Gate, which makes you think of the city of S. Francisco. A *symbol* is a sign that contains a rule in an abstract way: e.g. an algebraic formula. As relevant in communication (in thinking as well) gestures can be considered with respect to the linguistic and extra-linguistic modes of expression. The former is characterised as the communicative use of a sign *system*, the latter as the communicative use of a *set* of signs: "linguistic communication is the communicative use of a symbol system. Language is compositional, that is, it is made up of constituents rather than parts... Extra-linguistic communication is the communicative use of an open set of symbols. That is, it is not compositional: it is made up of parts, not of constituents. This brings to crucial differences from language..." (Bara & Tirassa, 1999; p. 5). In communicative acts the two modes co-exist. Students who learn the signs of mathematics, often recur both to their linguistic and extra-linguistic competences to understand them: for example, they use gestures and words as semiotic means of objectification. Typically, gestures are extra-linguistic modes of communication, whereas speech is on the linguistic side.

A NEW FRAMEWORK: THE PARALLEL AND SERIAL ANALYSIS

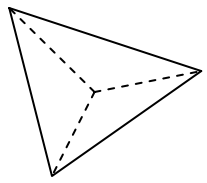

Figure 1

We show a brief example from the activity of some 8th grade students involved in approaching a geometrical problem, whose solution is a tetrahedron seen from an unusual point of view (in Fig. 1). Consider the following utterances by Gustavo, and his concomitant gestures.

Gustavo: Yeah, it is a solid, made of two triangles placed with the bases below, which are those starting in this way and going up, and two triangles with the bases above that are those going in this way [see Fig. 2]

Figure 2

We can analyse data like these in a double way, using what we call *parallel* and *serial* analysis. Both analyses take into consideration the dynamics of what we think as the major components of processes of objectification: not only speech and gestures (respectively ***s*** and ***g*** in Fig. 3), but also written words and mathematical signs (respectively, ***w*** and ***x*** in Fig. 3). The latter, even if not directly part of the communication acts, are a product of them, and often arise from gestures and words used by the involved subjects (Gallese, 2003; Sfard & McClain, 2002). Other components, e.g. drawings and artefacts possibly used by students, are beyond the aims of this paper.

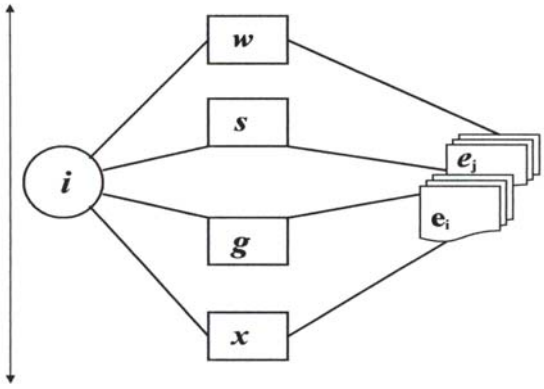

Figure 3: The ***PPO***

The components of objectification processes may develop according to two types of dynamics. We call the first dynamics *Parallel Process of Objectification (**PPO**)*; it results when (some of) the different components are seen as a bunch of processes synchronically developing (e.g. when one talks and gestures simultaneously). They can match or mismatch with each other in the way they are encoding information.

In such a case, we are interested in a parallel analysis of the components (see the vertical arrow in Fig. 3), which focuses on the mutual relationships among them, referred all to the same source ***i*** and possibly to different encoding $\boldsymbol{e_i}$'s. The main elements of a parallel process of objectification are: (i) the idea of semiotic means of objectification; (ii) the Information Packaging Hypothesis; (iii) Match and Mismatch (Goldin-Meadow, 2003).

We call *Serial Process of Objectification (**SPO**)* a second type of dynamics, which results when two different components are spread over time and happen in different moments, as steps of a unique process. An example is given by a sign produced as a frozen gesture (Vygotsky, 1997), or by a gesture embodying some features of a previous sign. In this case, we are interested in a serial analysis (see the horizontal

arrow in Fig. 4) focusing on the subsequent transitions from different sources ***i*** to different encoding e_i's.

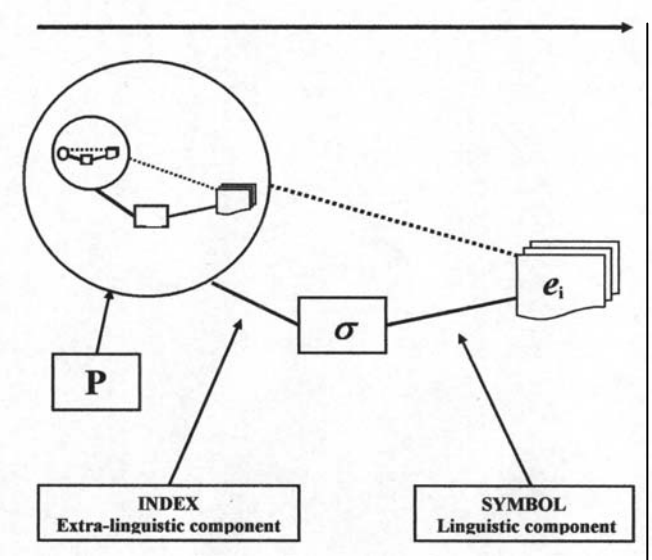

Figure 4. The ***SPO***

The Serial Process of Objectification is shown in Fig. 4. Its main elements are again: (i) the semiotic means of objectification; and (ii) the Information Packaging Hypothesis. But there are also two other elements: (iv) the indexical-symbolic functions of signs; (v) the linguistic and extra-linguistic modes of communicative acts. A serial process of objectification happens when one (or more) serial (or parallel) process(es) **P**, represented in the circle of Fig. 4, is (are) the grounding for the genesis of a new sign (indicated by σ).

For technical reasons, just one component appears in the circle, but there could be more. The sign σ is the pivot of the process; it can be any kind of sign: a drawing, a word, a gesture, a mathematical sign, etc. It is generated by the previous process(es) P and produces an encoding e_i of **P**. The relationships between σ and **P** are mainly extra-linguistic, whereas the relationships between σ and e_i are mostly linguistic. In other terms, the sign σ has an indexical function with respect to P, but it has also a fresh symbolic function with respect to the encoding e_i. Thus, the got ***SPO*** could be the basis for a new serial process, and so on, in an ongoing series of nested generalisations. Examples of ***SPOs*** are given by the learning of speech in kids or by that of reading written texts in young pupils. Mathematical examples are the processes undertaken by students who are learning Algebra or some other chunks of mathematical ideographic language, from Arithmetic to Calculus.

Generally both types of dynamics, ***PPO*** and ***SPO***, can support the genesis of signs. As a consequence, each process of objectification may be analysed from both points of view, that is as a parallel process and as a serial process. We call *parallel* and *serial* the two resulting types of *analysis*.

Let us go back to the initial example we can now interpret through the two analytical lenses. In the brief piece of the dialogue, Gustavo is trying to explain to the group mates the shape of the tetrahedron solving the task. Information at his disposal arises from his perception of the solid (he has in mind), and from theory he knows: 2D triangles and some 3D solids, according to which a tetrahedron is only seen as a triangular pyramid with a base 'down', not from the unusual perspective of the problem (see Fig. 1). The parallel analysis points out the conflict between the two pieces of Gustavo's theoretical knowledge concerning such 2D and 3D figures. Gustavo codes information he has through gestures, by some spatio-motoric features

conveyed in the motion of the hands. Hands and words both refer to the triangles constitutive of the tetrahedron in a match. The serial analysis shows that Gustavo's gestures (the hands frozen in the two positions, below and above) are mediating the transition from the 2D features of the triangles ("two triangles placed with the bases below", "two triangles with the bases above") to the 3D ones of the solid ("those starting in this way and going up", "those going in this way"). The serial analysis emphasises that Gustavo's gestures still have an iconic and indexical function in that their shape resembles their referents (the figures they express). But they are acquiring a symbolic function, since used as existing objects of a virtual geometric world and in relation with the genuine geometric objects. E.g. think of the metaphor of the "two triangles placed with the bases below […] and two triangles with the bases above". Or think of the contrast between the extra-linguistic, indexical modes in his speech ("... placed with the bases below... starting in this way and going up... those going in this way...") and the linguistic, symbolic ones ("... it is a solid, made of two triangles...").

After this episode, the experiment goes on and culminates with the acknowledgement by students of the tetrahedron as a triangular pyramid. Parallel and serial analysis allow to focus properly its dynamics. They are relevant to research from this point of view. In fact, parallel analysis reveals as a tool suitable for identifying conflicts, even before they appear to block or slow students' activities. On the other hand, the serial analysis represents a tool suitable for focusing the dynamics through which the subjects try to overcome obstacles met in their activities.

Acknowledgments

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