CURRICOLAR INNOVATION: AN EXAMPLE OF A LEARNING ENVIRONMENT INTEGRATED WITH TECHNOLOGY[1]

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An important question in considering the introduction of new technologies in mathematics curricola is that of their effectiveness in enhancing (or damaging) the real capabilities of students. To answer this question the paper sketches a theoretical framework, which frames the new technologies for mathematics as representational infrastructures: as such, they are analysed both as cultural semiotic systems and as cognitive energizers. The two concepts allow to define suitable adequacy criteria for testing the new technologies in the classroom. A teaching-learning environment integrated with technology is described as a concrete realisation of a technological-oriented Italian curriculum. An example of how learning can happen in this environment is described and a few final comments are drawn with respect to some questions asked in the Discussion Document of ICMI Study 17.

INTRODUCTION

The focus of this paper is on the theme 4 of the Discussion Document. To do that we introduce a theoretical framework suitable to test how the environments integrating digital technologies are adequate for the effectiveness of maths learning (§1). Then we exemplify this definition illustrating a concrete learning-teaching environment where the learning experiences take advantage of the affordances supported by the new technologies (§§ 3, 4). The environment is developed within a concrete technological integrated Italian curriculum, which is sketched in advance (§ 2). In the end (§ 5) we discuss how our theoretical framework and the methodologies illustrated in the examples are useful in understanding the impact upon the teaching/learning of mathematics.

1. THEORETICAL FRAMEWORK

The introduction of Information and Communication Technologies (ICT in brief) in mathematics curricula has been stressed and encouraged in these last years. However the benefits from such an introduction are neither necessary nor automatic; the matter must be considered carefully, looking at its many aspects and possible negative effects. In fact, as shown in Artigue et al. (2001), much of the theoretical and empirical research dealing with the use of ICT in mathematics education is prevalently concerned with the added-value component provided by the technology and rarely faces critically an approach to ICT based on an ecologically sustainable use. A major disadvantage consists in the fact that papers on ICT are more concerned

[1] Research program supported by MIUR and by the Università di Torino and the Università di Modena e Reggio Emilia: Significati, congetture, dimostrazioni: dalla ricerche di base in didattica della matematica alle implicazioni curricolari (PRIN n. 2005019721).

in “learning how to use technical tools incorporated in the computer ... than [in] understanding the theory behind those tools” (*ibid.*), namely in what can be called a *blind use of technology*. This often corresponds to *a-critical practices* in the classrooms, which are limited to control the functioning of the tool, which is “insufficient for a successful mathematics outcome” (Thomas & Hong, 2004).

What is needed is an *aware use of technology,* which means to understand if, how and when the technological artefacts can mediate/support/carve the construction of the student’s mathematical knowledge in the classroom. To do that, one must consider ICT’s in a wider setting, namely to analyse them from a multiple perspectives: that is, from a *didactic* point of view (e.g. considering the role of the teacher, of social interactions induced by the used technology, and so on); from a *cognitive* point of view (e.g. considering how technology changes the mental structures of the learners); from a *cultural* point of view (e.g. considering the framework of rationality towards which the use of technologies may push the student). An interesting analysis in this direction has been developed by J. Kaput *et al.* (2002), who introduced the notion of *representational infrastructures* (= RI):

> The appearance of new computational forms and literacies is pervading the social and economic lives of individuals and nations alike.…The real changes are not technical, they are cultural. Understanding them… is a question of the social relations among people, not among things. The notational systems we use to present and re-present our thoughts to ourselves and to others, to create and communicate records across space and time, and to support reasoning and computation constitute a central part of any civilization’s infrastructure. As with infrastructure in general, it functions best when it is taken for granted, invisible, when it simply “works”.

The challenge today is how to design learnable systems within suitable and up-to-date representational infrastructures. Roughly speaking, the major point consists in seeing how the ICT involved in a system fit within RI according to the following two criteria: a) as *Cultural Semiotic Systems*; b) as *Intrinsic Cognitive Energizers.*

Cultural semiotic systems (Radford, 2003) are those systems which make available various sources for meaning-making through specific social meaningful practices; such practices are not (only) to be considered within the strictly school environment but within the larger environment of the whole society, embedded in the stream of its history. For example, Kaput & Schorr (2002) claim that the development of algebra in the history of mathematics was made possible by an entirely new mode of thought “characterized by the use of an operant symbolism, that is, a symbolism that not only abbreviates words but represents the workings of the combinatory operations, or, in other words, a symbolism with which one operates”.

Intrinsic Cognitive Energizers are here defined as systems which make available varied sources for meaning-making through specific cognitive meaningful practices; such practices do enter intrinsically in positive cognitive resonance with the subjects, because of cultural and biological reasons. Examples are given by the quick and mainly visual interaction with the information provided on the computer screen in computer games and in access to all kinds of information provided by the internet system.

In the next two paragraphs we shall illustrate this framework introducing a concrete teaching-learning environment developed within a technological integrated curriculum of mathematics. We'll sketch the latter in next paragraph and discuss the former in the successive § 3.

2. AN EXAMPLE OF TECHNOLOGICAL INTEGRATED CURRICULUM: THE UMI[2] PROPOSAL AND THE *MATHEMATICS LABORATORY*

Modern society requires massive use of mathematical knowledge and skills. A significant act with this respect is the UNESCO resolution in 1997, that underlines how "mathematics education has a key role, in particular at the level of primary and secondary school, in the comprehension of mathematical concepts and in the development of rational thinking" (http://www.unesco.org/science/physics.htm).

The UMI proposal of mathematical curricular innovation is inspired by the UNESCO resolution and, in particular, take into account both the instrumental and the cultural functions of mathematics. The essential core of UMI curriculum provides a foundation for pupils' mathematical competencies through 4 content areas (*numbers and algorithms; space and shapes; relations and functions; data handling and previsions*) and 3 processes areas (*argumentation, conjecturing, proving; measuring; solving and posing problems*). The 7 areas are essentially the same for the whole pre-university school from 6 to 19 years. The teacher is supposed to tackle these themes in an integrated manner, trying to connect them to other topics and to other subject disciplines. The UMI curriculum ([3]) contains also some reflections on the *Mathematics Laboratory* that are very important for the scope of this paper. In fact, the *Mathematics Laboratory* is conceived as a teaching-learning environment based on the use of instruments and aimed at the construction of mathematical meanings. It is not intended as a physical place other than the classroom, but as a structured set of activities with the goal of building the meaning of mathematical objects. As such, it involves people (students and teachers), structures (classroom, instruments, organization of space and time) and ideas (plans for didactical activities, experiments). In this sense, it passes the two adequacy criteria, stated in § 1. This is shown also by a metaphor introduced in the UMI document, where the *Mathematics Laboratory* is compared to what happened in the *Bottega d'Arte* of Renaissance artists, where the novices learned through a *cognitive apprenticeship*, namely by doing and watching what was done by experts, communicating with one another and with the experts, who pointed out the cognitive difficulties that the newcomers would encounter. The construction of meanings within the *Mathematics Laboratory* is strictly linked to the instruments used when carrying out the given activities and to the interactions among the participants in the activities (Bartolini et al., 2004). It is important to remember that tools are the result of a cultural evolution: they have been produced with specific aims and, as RI, represent ideas in a culturally shared and technologically up to date way. This has important didactical implications: first of all, meanings do not live only in the tools and cannot emerge purely from the interaction

[2] *Unione Matematica Italiana (Association of mathematicians and mathematics Italian teachers)*
[3] The curriculum is available at: *http://www.dm.unibo.it/umi/italiano/Didattica/ICME10.pdf*

of the pupils with the tools. Meanings are rooted in the aims for which the tools are used as culturally shared artefacts, and in the strategies related to the use of those tools that are elaborated in the course of the activities, which make palpable their cultural content. Moreover, the appropriation of the meanings requires individual reflection on the objects of study and the proposed activities. The construction of meaning is strictly linked to the communication and sharing of knowledge in the classroom, through collaborative or cooperative group work and through the *mathematical discussion* orchestrated by the teacher.

3. A CONCRETE EXAMPLE OF A TECHNOLOGICAL INTEGRATED TEACHING -LEARNING ENVIRONMENT

The UMI document has also many examples of teaching activities; however it does not contain indications on specific didactical paths that suggest how to develop its contents. How its curricular indications can be transformed into structured and concrete didactical paths in the classroom is an affair that has been left to the schools and to the teachers, in accordance with the Italian curricular tradition. For this reason a group of upper secondary school teachers[4] has designed a didactical plan from the curricular indications of the UMI proposals and according to the educational framework illustrated in §1. The result has been a teaching-learning environment, where the ICT are integrated as RI, called *Mathematics in the Web* (indicated as MW-project in the following).

The main features of this environment are sketched in the following points:

a) using ICT's to design, to build and to realize mathematical activities, which *make sense* (*attività sensate* in Italian); the Italian word is used with three different meanings: reasonable (that is attentive to the specific possibilities and constraints of the class); linked to the natural abilities and particularly to perception; ruled by the intellect and specifically by a theory (the last two meanings were used by G. Galilei to feature what he called *sensate esperienze*);
b) using a long-term didactics (contrasted to short term educational projects), with particular attention to the construction of the meaning of the mathematical concepts and to the development of a critical thought in the students, considered essential for citizens' awareness in nowadays society;
c) engaging students in explorations, building, scaffolding, communicating activities, so that that the teacher can have information not only about their products but also about their thinking processes;
d) gradually introducing the students to theoretical knowledge (different from the factual one), where such questions like "why does it work?" make sense and where the answers give reason not only for links among related facts but also in terms of logical consequence between the phenomena that one is trying to explain and the statements that are the foundations of the theory within which the facts are framed.

[4] Ercole Castagnola, Cristiano Dané, Michele Impedovo, Domingo Paola, and others. All the materials (in Italian) are available at the following address: http://www.matematica.it/paola/Corso%20di%20matematica.htm .

As far as the specific goals of the project are concerned, there are three main conceptual areas, which are developed according to the UMI curriculum:
(i) beginning *probabilistic thinking*; (ii) beginning the study of *changing quantities*: specifically modelling activities of phenomena that evolve and change in time, in order to describe them and to foresee their evolution; (iii) fostering students *spatial intuition* in all activities where they use exploring, observing, finding abilities.
We shall make some comments only on point (ii), which is particularly akin to the topics discussed in this paper. These activities concern the context of change and of movement, within the core area *Relations and functions* in UMI curriculum and approach some of the basic concepts of Calculus: from functions as modelling tools for phenomena of changing quantities to derivatives and integrals. The project uses systematically a variety of different ICT devices which incorporate suitable different aspects of change & motion phenomena (e.g. motion detectors connected to computers). As widely discussed in Kaput et al. (2002) such devices are genuine new representational infrastructures, which can produce a positive cognitive resonance in pupils and support their learning. There are many projects in the world that are developed according to this philosophy, e.g. *Simcalc, Playground, Data Capture, WebLabs1project*. The use of ITC for the mathematics of change is a good example of a teaching design which fits very well the adequacy criteria a) and b) stated in §1. Essentially the concept of function is approached within a very powerful RI, where cultural and cognitive aspects are in deep resonance.
Specifically, the MW-project is based on some ideas by D. Tall on the approach to Calculus, which are widely sympathetic to the general philosophy discussed previously. In fact the approach to the main concepts of Calculus is built up starting from the three following fundamental *cognitive roots* (Tall, 2000): the notion of local straightness as a cognitive root for differentiation; the idea of a graph that "pulls flat" when it is stretched more horizontally than vertically for the mathematical concept of punctual continuity; the notion of area under a graph of a continuous function and the graph that "pulls flat", for the relationships between integration and differentiation.
The project develops such concepts using the software TI-InterActive!, which works as a generic organizer, in the sense of Tall (Tall, 2000). Such a software is an interesting example of an ICT, which fulfils the adequacy criteria of §1. In fact, it collects the functions of different products into a unique environment: a numerical, graphical and symbolic calculator; a word processor; a spreadsheet; the possibility of importing data from different environments, specifically from graphic-symbolic calculators and from probe devices that get measures of physical quantities; a browser to navigate in the web and to interface with other environments and software (in § 4 we shall show an example where the students use the software Graphic Calculus). The project uses TI-InterActive! to design work-sheets in its teaching-learning environment for exploring activities that foster the production of conjectures by students, as well as their validation. In all this work the activity of the teacher is essential to coach all the different processes. Successively, the teacher supports students in structuring and scaffolding the learnt concepts within a theory. In the end

the work-sheets can mediate part of the theory, which organizes and systematizes the learnt concepts and the algorithms in a coherent framework.

4. AN EXAMPLE OF A TEACHING EXPERIMENT

To give an idea of the type of learning that happens in such an environment, we shall present some excerpts from the protocols of students who are approaching the concept of the derivative dy/dx as the limit of the rate of change Δy/Δx. The excerpts illustrate in an emblematic way how the interaction with Graphic Calculus, suitably coached by the teacher, can support conceptualisation processes in pupils. In the work-sheet, which is built with TI-InterActive!, the students are required to explore *Gradient*, a modulus of Graphic Calculus and, successively to write their results in the TI-InterActive! page. The space does not allow to enter into details and we shall limit ourselves to some spots. The students are in the 11th grade of a scientifically oriented school, participate to the MW-project and are introduced to the fundamental concepts of *Calculus* since the beginning of high school (9th grade). They are used to work in small groups and to participate to collective discussions orchestrated by the teacher. As said above, they are also accustomed to use technological devices, e.g. sensors to investigate motion experiments. In the example, they must study the graph of the function $f(x) = 0{,}5\,x^3 - 5x^2 + 3$. As shown in Fig. 1, the software generates: the graph of the function; its tangent, which moves dynamically along the graph, while a point traces it; the graph of the slope value of the tangent, while it is moving.

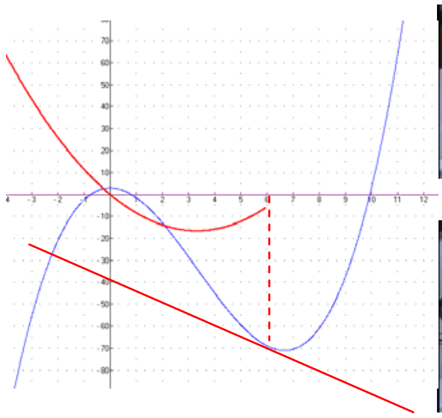

Figure 1

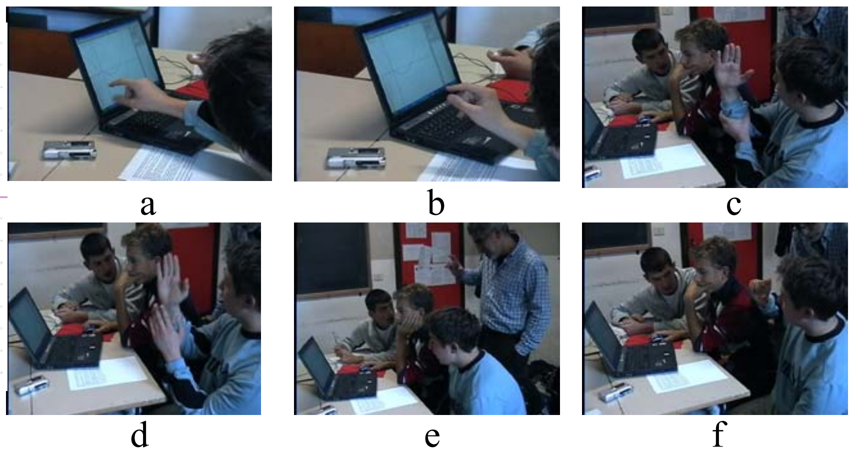

Figure 2

Fig. 2 shows some emblematic steps in the genesis of the rate of change concept. It is precisely the interaction with the software to generate in the students the first germs of the relationships between the rate of change and the slope of the tangent. In fact they have produced the graph of a secant to the graph of the function, which joins couples of points on the graph whose abscissas differ for a constant value Δx, determined by the students themselves. They fix successively Δx equals to 0.1, 0.01, 0.001, and so on, so that the secant becomes a quasi-tangent (Fig. 1). It is interesting to observe in the pictures of Fig.2 how the gestures of the student (more than his words) show the way he is acquiring the rate of change notion.

Stud1: This straight line must join [Fig 1a], ok, the X interval...it is [always?] the same [Fig 2b],

Teach.: The X interval is the same; delta X [Δx] is fixed

Stud1: Delta…eh, indeed, however there are some points where to explain it, one can say that this straight line must join two points on the Y axis, which are farther each other hence it is steeper towards... [Fig 2c]

Stud1: Let us say from here, when, here, when however it must join two points, which are farther, hence there is less distance [Fig 2d] [...]

Teach.: is it decreasing? [Fig 2e] [...]

Stud1: They are less and less far; in fact we can say that the slope is going towards zero degrees.

Teach.: Uh, uh

Stud1: Let us say so

Stud2: Ok, Δy over Δx at a certain point here it reaches points, ...

Stud1: The points are less and less far [Fig 2f] [...]

Teach.: What does this object represent, when h represents this distance, this small interval? ...

Stud3: No, It is neither a tangent, it is a ... secant...the more I make this small, when I have h very small, then it represents the slope of the tangent in that point, exactly in that point and hence... [...]

Stud2: It is something, which is useful to determine the slope in X; since we cannot do it directly; that is we need the X + h, which when multiplied by a certain number of things , you see it through computations, then it goes to zero; so we can eliminate it and we have ...

It is a dynamical idea that contains: the limit process with Δx that becomes smaller and smaller (Fig. 2 a, b), the relationship between the slope and the rate of change (Fig. 2c); the relationship between the Δx (here constant) and the Δy (Fig. 2 d, f).

The data of our teaching experiment, with all their limits, confirm that this approach to ICT is an useful research tool to understand the ways in which technological artefacts can support the construction of the student's mathematical knowledge. In fact, the analysis of the cultural and cognitive ingredients of the ICT used in the classroom allows to consider the added-value component provided by the technology not limited to its purely technical features. In particular, the analysis of ICT according the two dimensions (cultural and cognitive) stresses students' learning that happens in such an environment: a rich interplay between the perceptuo-motor and the symbolic-reconstructive learning, as discussed in Arzarello *et al*. (2005).

5. SOME CONCLUSIONS

Our example and the whole MW-project illustrate how the mathematical content and the methodological issues of a curriculum must take into account suitably the rich resources made available by RI. It is the same concept of mathematical literacy to change within such new technologically integrated environments. A major goal of today math education consists in contributing, together with all other subject disciplines, to the cultural development of citizens, in order to enable them to take part in the social life with awareness and a critical eye. The competencies required for a citizen, to which mathematics education can contribute, include for example: communicating information appropriately, perceiving and imagining, solving and posing problems, planning and constructing models of real situations, making choices in conditions of uncertainty. These goals are much more ambitious than the old ones, like making computations and remembering a corpus of knowledge, which was supposed essential for a mathematical culture. The new literacy requires a strong mathematical experience and the habit of working with mathematical objects and within mathematical environments integrated with the new ICT. As such, ICT are specific for mathematics but share a lot of common features with other RI.

The main design principles according to which the new ICT are to be articulated, should satisfy a twofold principle. From the one side, they should make available rich and multimodal experiences to approach mathematical conceptualisation within a rich and meaningful RI. From the other side, the experiences made in such environments should be 'easier' and more motivating for the students because of cultural and cognitive reasons. In fact, concepts should be tackled starting from their cognitive roots and the environment should represent them in a powerful dynamic and interactive way, so that it can support and enhance a perceptuo-motor approach to them, namely an approach that involves massively action and perception and produces learning based on doing, touching, moving and seeing (like in the excerpt of our example). But the ultimate rationale of any ICT remains the teacher: it depends heavily upon her/his teaching project if the students will be satisfied, provided they know simply *what* to do or *how* to do something, or on the contrary if they will ask themselves also *why* things they have found or they are using are effectively so.

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