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Experimenting and explaining quantity variations to learn functions with Cabri-Gèomètre

Summary

Nowadays in Mathematics Education, thanks to the modern available technologies, it is possible to introduce early at school themes and problems dealing with the mathematics of change and variation, which before were only tackled in the last years of secondary school. We present an activity in Cabri-Géomètre, to introduce students of lower secondary school to the mathematics of change and variation. We analyse this activity, particularly students' cognitive processes, using a theoretical framework constituted by three essential elements: some research outcomes relative to the use of a Dynamic Geometry Software also for teaching branches of mathematics different from geometry; the embodied approach to mathematics particularly for the attention given to metaphors as the essential mechanism in human cognition; the instrumental approach, recently used in Mathematics Education, based on the distinction between an artefact and an instrument.

Introduction

The Italian traditional approach in teaching-learning Calculus, at secondary school level, has always been focused on the formal and syntactic aspects, based on symbolic calculation and manipulation as necessary pre-requisites. For this reason, the specific topics of Calculus are inserted in upper secondary school. However, the mere simplicity of tracing a graph or manipulating an algebraic expression given today by new technologies makes us reconsider in a

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new way such a traditional approach, which has been one of the most powerful means of student selection in math courses.

The software made available today allows us to re-structure and re-organise the teaching-learning of Calculus, regarding the sequence of topics, and the school level in which they can be introduced. We have also to ponder on how the meaning of the mathematical objects can change with the use of software.

In mathematical activities with tools there exists a strict relation between practices and meaning: being the practices dependent from the tools used, then there exists a strict relation too between the meaning of the mathematical objects and the tools used for coping and solving problems (Paola, in print).

We think that the topics of Calculus could be considered as conceptual knots, the most entailed in educational change, which technologies today render possible. The studies of David Tall are a perfect example in order to highlight this point. We can think, e.g. to the role played by new technologies in Tall's papers, for grounding the fundamental concepts of Calculus on the cognitive roots (Tall, 1989; 2002) and for managing the teaching-learning activities on them.

Another cognitive and pedagogical advantage in the use of technologies consists in the possibility of approaching the mathematical concepts in a dynamical way. This has been pointed out by many researches in DGS. In particular the problem has been studied by Kaput, who presents three different kinds of consequences determined by the use of technologies in the teaching-learning of Mathematics: the shift from a static knowledge to a dynamic one; new modalities of data representation (obtained through simulations or by specific devices as sensors) and of data patterns and relations; the computation of difficult problems, such as discrete non-linear dynamic systems, where powerful technologies are necessary (Kaput, 2002).

In this paper we want to comment on the ways in which a tool as Cabri-Géomètre allows meaningful activities for the study of quantity variations, before having at disposal such symbolic manipulation techniques, (the same that were considered essential for an approach to Calculus). We think that an earlier approach to Calculus, at lower secondary school, could improve and enhance the competences related to exploration and observation, on which the construction of meanings of Calculus concepts can be based later, at upper secondary level.

The possibility of doing activities for an earlier and informal, but meaningful approach to Calculus at lower secondary level, offers an opportunity to have easy access to fundamental concepts, very important in society nowadays.

Theoretical frames

The elements we use for a theoretical framework are of different kinds:

1. Some outcomes of the last years' research, relative to the use of a Dynamic Geometry Software for learning geometry, are used in a wider perspective, in order to enlarge the teacher's purposes to look at the same DGS also for teaching other branches of mathematics, as pointed out by Laborde & Mariotti (2001);
2. Some recent studies in the field of cognitive linguistics, namely the embodied approach to mathematics, as described by Lakoff & Nùñez (2000);
3. The instrumental approach, coming from ergonomics and recently used in Mathematics Education, which is based on the distinction between the object (artefact) used by a subject and the object together with the kind of use activated (instrument) (Rabardel, 1995; Verillon & Rabardel, 1995).

According to Kynigos (in Nemirovsky et al., 2004), within the first point, many papers in the last years examine the role of constructions in geometry, focussing on them as a means to define and think with properties of a geometrical figure, rather than perceiving it as a shape or a drawing (e.g. Mariotti & Bartolini, 1998; Mariotti, 1996). Others examine different dragging modalities, as sets of continuous change under the constraint of preserving the properties used in the construction of the figure. Some of them distinguish between different modalities of dragging, according to the kind of activity made by students and the kind of feedback furnished by the software (e.g. Arzarello et al., 2002); others highlight the importance of experimenting with wandering dragging to appreciate the initial problem and to subsequently test specific conjectures (Furinghetti & Paola, 2003). Some studies concentrate on the role of trace and locus as tools in the learning process (e.g. Laborde & Mariotti 2001). A particular set of researches points out the importance of measurements in the use of a DGS, because they are digital representations of values of objects and relations (angles, segments, areas etc), in the sense that they change in conjunction with dragging (Olivero & Robutti, 2001; 2002). All these approaches focus on the fundamental role of those different kinds of use of a DGS: the introduction of students to proof, through various activities of construction, exploration, production of conjectures. In them, proof is seen as a long-term process, supported by those various activities mentioned before, and DGS is seen as a supporting tool for the transition from perceptual to theoretical perspectives (Arzarello et al., 2002; Healy, 2000). Framed in this transition, a group of reports have stressed the importance of teacher mediation (Jones, 1997; Olivero and Robutti, 2002; Gardiner and Hudson, 1998).

Recently, some studies are specifically aimed at using a Geometry software in order to teach some topics of algebra, such as symbolic expressions, equations, functions, and so on (e.g. Laborde & Mariotti, 2001; Cha and Noss, 2002; Kynigos and Psycharis, 2003). Our aim is to introduce students to theoretical thinking, starting from explorations in a DGS as Cabri, in which they can experience the variation of quantities in order to construct a meaning for functions, their graphs and their critical points.

The second element of our theoretical frame consists in the fundamental cognitive structure introduced by Lakoff and his colleagues: metaphors, as essential mechanisms in human cognition. Recently, Lakoff and Nùñez (2000) have extended this argument to a detailed account on how mathematical cognition is first rooted in our body via embodied metaphors, then extended to more abstract realms through the so-called *conceptual metaphors*, i.e. inference preserving mappings between a source domain and a target domain, where the former is more concrete and better known than the latter. They show how mathematical cognition is built on the same mechanisms of our general linguistic and cognitive system. The embodied approach reveals useful information in describing students' cognitive evolution within technological environments and for designing suitable teaching experiments. It shows a basic unity in their cognitive evolution from perceptions, gestures, and actions, to more theoretical aspects (for an example, see Arzarello, 2004, in print). Embodied cognition is also useful to analyse the dynamics of the social construction of knowledge by pupils: specifically the metaphors, introduced by students in a group or class discussion, or by the teacher when she/he wants the students to concentrate on a particular concept or to construct a new one (for an example, see Robutti, 2003), reveal powerful tools for supporting and sharing new ideas.

The third element of the theoretical framework is the instrumental approach, introduced by Rabardel (Rabardel, 1995; Verillon & Rabardel, 1995) to clarify the effect of technical devices on learning processes. In this approach, the technical devices are considered with two interpretations. From the one side, an object has been constructed according to a specific knowledge, which assures the accomplishment of specific goals; on the other side, a user interacts with this object, using it in different ways. So, the object in itself is called an *artefact*, that is, the particular object with its features, realised for specific goals, and it becomes an *instrument*, that is, the artefact with the various modalities of use, as elaborated by an individual, who is using it. The notion of artefact refers to the object, with its characteristics, while the notion of instrument is referred to the subject who uses the artefact, with particular modalities, related to a specific task. So, the instrument is conceived as the artefact together with the actions made by the subject, organised in collections of operations, classes of invariants, and utilisations schemes. The artefact, together with the actions, constitutes a particular instrument: so, different subjects can have different instruments using the same artefact, or the same subject can use the same artefact as different instruments.

For example: the rule is an artefact, which can be used by a student to trace a line, as the locus of aligned points in a plane. The transformation of the artefact into an instrument is made through the action of putting it on the paper, and tracing a line following one of its sides. The same artefact, however, can be used by a driver on a map, to control and measure distances between two points.

The transformation of the artefact into an instrument is made through the action of pointing it at a point, and reading the distance between that point and another. Therefore, the artefact becomes a different instrument through the purpose of the actions involved in by the subject.

As different and coordinated utilisation schemes are elaborated successively (by the subject, with her/his actions), the relationship between the artefact and the subject can evolve, causing the so-called process of *instrumental genesis* (Guin & Trouche, 2002) revealed by the schemes of use (the set of organised actions to perform a task) activated by the subject. In principle, it is not assured that this evolution is consistent with the original purpose for which the object has been designed. While the artefact is an object that can be considered static, in the sense that it does not change its features in time, the instrument can be thought dynamic, in the sense that it can change its features, according to the schemes of use activated by the user.

In our study, the subjects are the students, who use Cabri in different ways, with various actions and commands. The process of instrumental genesis can be integrated with the construction of knowledge, because the students, solving a task with Cabri, do not only press the commands or move the mouse in order to obtain a result (whatever it may be), but they must also control it, interpret it correctly and use it in their conceptual path to the task.

The integrated use of these three elements allows us to analyse the teaching-learning activities from a wide perspective, giving rich and various ways for interpreting them. To use a DGS as Cabri in the introduction of some topics of pre-Calculus is a challenge we are trying to carry out, just to explore potentialities and pitfalls of such an integrated approach. The first outcomes we obtained encourage us to go on this way deeper and deeper, and it is exactly what we are doing, following the same class of students in their curricular path over the years.

The teaching experiment

This activity has been proposed and carried out in two classes of 9th grade of a scientific school with the experimentation in computer science (P.N.I., Piano

Nazionale per l'Informatica). In one of them (class A), when the problem was proposed, the students were at the beginning of their use of Cabri-Géomètre. They had used it only for a short time, especially in activities aimed at the construction of figures and the exploration of geometrical properties.

In the other class (class B) the students were more skilled in using Cabri, having utilised it in various activities aimed at exploring geometrical problematic situations, not so simple for their age and their knowledge.

Both in class A and in class B, the students worked in pairs (in only one case a group of 3 was necessary) in a computer science laboratory, with the task of describing on a piece of paper the applied strategies, needed to solve the problem.

The activity of a pair of students in class A was video-taped.

The text of the problem is ordinary:

Consider a set of rectangles with a fixed perimeter, for example at 12 cm. What can you make out of their area?

In class B, the text of the same problem has been slightly different, due to the aim:

What can you make out of rectangles with a fixed perimeter?

Of course, the open text of the problem was on purpose: in fact, the open problems (Arsac, Germain & Mante, 1988)[3] naturally guide the students in exploring and observing, and so they foster the production of conjectures. Moreover, the open problems motivate students at justifying the conjectures, particularly when they are solved in a DGE as Cabri, in which the observation of changes and invariants is increased.

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[3] The features of an open problem are the following:

- it has a brief text
- it does not force a method or a particular solution
- it concerns a conceptual domain with which the students are familiar with.

The text of the problem proposed in class A is less open than the one assigned to class B. This choice is mainly due to the fact that those students were at their initial experiences with the use of Cabri: a formulation of the problem suggesting the object to explore and to find seemed more reasonable.

In both cases the work-sheet, as it is represented in Figure 1, has been prepared by the teacher in this initial phase, to avoid an excessive attention of the students on the technical details of the construction, instead of the mathematical concepts involved in the problem (see the Cabri II - file "rettisope.fig")

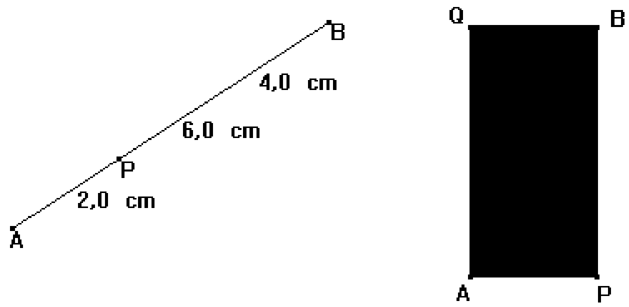


Figure 1

As the figure suggests, the segment AB represents the half-perimeter of the rectangle. The point P, which has been constructed on the segment AB, and so is bound to it, determines the two segments AP and PB that are the two sides of a rectangle of perimeter 12 cm.

Once having constructed the segment AB, fixed a point P on it, constructed the two segments AP, PB and determined the measures of AP, PB and AB, one can construct the black rectangle with the following actions:

a) to draw the ray with origin A, on which to transfer the measure of AP

b) to draw the perpendicular line to AP by point P

c) to draw the ray with origin P, on the perpendicular line drawn at b) then to transfer the measure of the segment PB on it

d) to draw the perpendicular line to PB by point B

e) to draw the perpendicular line to AP by point A

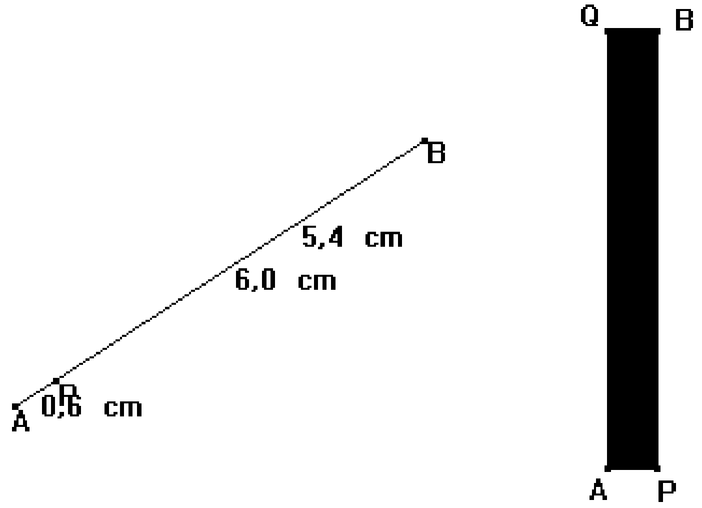
f) to mark point Q, intersection point of the two straight lines, drawn at d) and e)

g) to construct, in the menu “polygon”, the polygon APBQ that is a rectangle, because of this construction

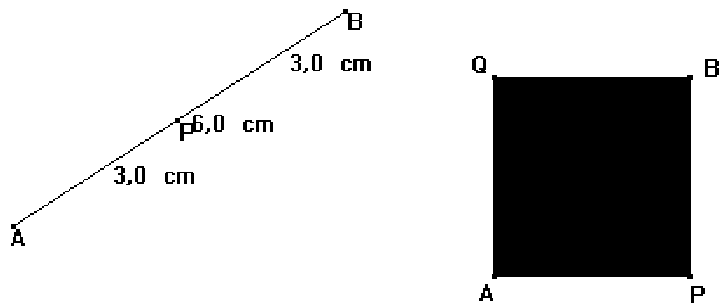
h) to hide all the objects, except the segments AB, AP and PB and the rectangle APBQ

i) to colour the rectangle with the tool “fill”.

At this point the student, invited to drag the point P on AB, looks at the changing rectangle, which maintains a fixed perimeter, while its area is changing and it reaches the value 0 in the limit cases in which P coincides with A or with B.



B
5,4 cm
6,0 cm
P
A
0,6 cm
Q
B
A
P



B
3,0 cm
P
6,0 cm
3,0 cm
A
Q
B
A
P

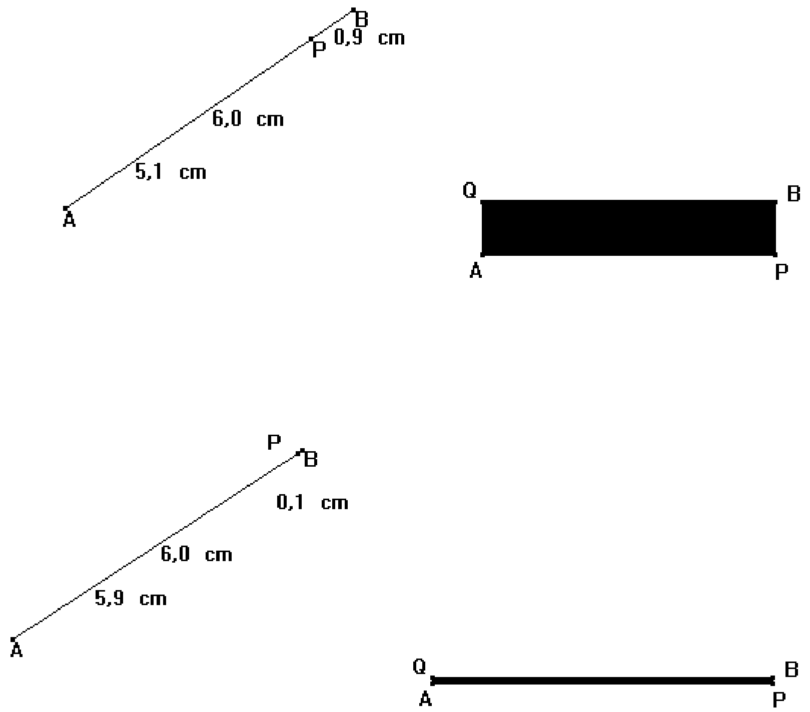


Figure 2

The first observations have been made at a pure perceptive level. The students just observe that *if the point P moves, than the rectangle moves.* The passage from a perceptive level to a variational level (consisting in the observation that the area, or other quantities of the rectangle depend on the position of P, and so on the length of AP, while the perimeter does not depend on the position of P) is subsequent. It has been obtained with modalities different from one student to another, even according to the experience he has reached in working with Cabri and to the more or less open formulation of the problem.

For example, in class A the explorations have been immediately concentrated on the area, while in class B they also deal with the measures of the sides, sometimes of the diagonal, often of the angles as invariants.

The students of class A at the beginning were more static in their explorations than their mates of class B: from the deep analysis of the videotaped session, the teacher observed[4]: "While Alessio is dragging point P, the rectangle is changing its shape, and Ettore says: *Let's start from the hypothesis that all the rectangles had the same area.* Alessio goes on dragging and Ettore writes on his exercise book $A = l_1 * l_2$ and attempts with different numbers. Only after attempting with the numbers, he realises that the area changes (moreover, he asks the teacher for a calculator in order to do some trials and Alessio, who often abandons dragging for looking at Ettore's work, supports his request). They work a lot on the paper, before dragging again. They discover the cases with area equal to zero in the numeric exploration made on the paper. It seems that they drag principally for checking and verifying conjectures just made, or looking at particular cases, than for observing regularities in a dynamic way".

In class B, however, the use of dragging is more evident and dynamic, as we can expect from students who can use such a powerful tool. This different behaviour is a typical of the students of these two classes and also of the strategies they use to justify that the square is the rectangle with the maximum area.

This fact suggests that the evolution of Cabri from an artefact to an instrument with particular schemes of use (those of dragging as means of exploration, observation, conjecture and check) is not natural, but it needs a lot of time and a continuous and conscious support of the teacher. Besides, the instrumental genesis, namely the process by which a software (artefact) becomes a instrument for doing mathematical activities, depends not only on the software constraints and potentialities, but, more generally, on the didactic

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[4] Domingo Paola, one of the two authors.

environment (students' knowledge and expertise, institutionalised knowledge which the teacher refers to). For this reason the instrumental genesis is a process that the teacher must be aware of, in order to provide meaningful learning activities for the students.

The first observations made at a mere perceptive level can be enriched by the fact the measurements of area and perimeter enter the scene. In class B some students autonomously observed, using the tool 'animation'[5], that the numbers for the area measures do not vary uniformly, but they change more and more rapidly, as point P is moving from the centre of segment AB to one of the two end-points A or B. As we can see, Cabri offers the possibility to look simultaneously at the numerical and geometrical changes and this favours the production of linking metaphors between two mathematical domains. According to Lakoff and Nunez (2000), linking metaphors are conceptual metaphors which conceptualise one domain of mathematics in terms of another. In this case the students link the rectangle variation to the number variation (and also to the rate of this variation). This fact let the students understand the main features of the mathematical relation between the area and the length of AP, namely the functional dependence of the area of the rectangle from the length of AP.

In class A, it was the teacher who suggested to the students the use of 'animation' and the observation of area measures while the animation goes on. This direct intervention of the teacher points out the importance of the theoretical thinking (the lens of the theory) also to see what we are looking at. Without the teacher intervention, in the class A, where the instrumental genesis has not happened yet, the students are not able to see what they are looking at, but after the teacher intervention they become able to link the numeric variation to the change in the figure.

[5] This instrument allows us to connect a point with one or two degrees of freedom with a spring that enables it to move. In doing so, the student may observe, without the need of using the mouse, how the figure varies, once the animation has been activated. In this case the students put the spring on point P, movable on the line AB.

However in both cases, finally the observation of the *way* in which measures change[6] is shared and thought significant by all the students of both classes.

We observed, in both classes and above all in B, students tracing gestures in the air, representing the variation of area. Only in one case such gestures were transferred onto paper without an explicit request of the teacher. Some of those students, but also others who did not show clear ideas on the variation of the area, used the instrument of finite differences[7], in order to search for a pattern in the numerical values of area (or to check the conjectures).

Most of the students who took part in considering the first differences used a table like the one shown below.

AP	PB	Area	Δ Area
0	6	0	
1	5	5	5
2	4	8	3
3	3	9	1
4	2	8	– 1
5	1	5	– 3
6	0	0	– 5

Table 1

Even if the table contains only integer values expressing the measures of the rectangle sides, it suggests that the area decreases less and less, while the length of AP varies from 0 to 3, and decreases more and more while the same length varies from 3 to 6. Two pairs of students of class B tried to consider also decimal numbers of measures, which is possible in Cabri, but they had some difficulties for the approximation of the measurement tool (using two decimal digits the difficulty

[6] The two information are both important: to discover that a quantity changes and also to say *how* it changes, i.e. if it increases or decreases and *how* it increases, and if it increases, it can do it more and more or less and less.

[7] In the previous lessons, the students of both classes have learned to use the method of the first differences in order to understand if a function is increasing or decreasing and how.

consisted in moving P in so that the measure of AP could vary with a uniform rate. With only one decimal digit the difficulty consisted in a low sensibility of the measurement tool, which did not let possible to link a unique measure of the area to every measure of AP).

A pair of students of class B utilised a spreadsheet, writing in the first column the decimal numbers from 0 to 6, changing with a step of 0.1; in the second column the formula =6 – A1; in the third column the formula = A1*B1 and in the fourth column, from the second cell, the formula = C2 – C1. In doing so, copying the formulas in the proper way, they obtained a table of the same kind of Table 1, with measures with one decimal digit. In this situation the students used the graphic environment of the spreadsheet, in order to have the parabola representing the function area.

At this point of the exploration phase, every student was sure that the maximum for the area was in the case AP = 3, i.e. in the particular case of a square; most of them argued the graph shape of the function area and some of them made some considerations about the symmetry of that graph.

A pair of students in class B made a drawing similar to Figure 3 (it was a sketch, of course).

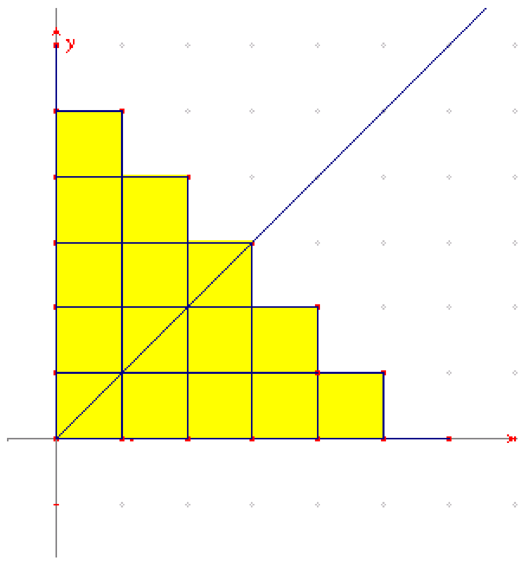

Figure 3

The various approaches described above are characterized, from a cognitive point of view, by a shift from perceptual to more theoretical aspects, linking together in different ways numerical, graphical and symbolic results.

We can say that the knowledge about the solution of the problem was diffuse and grounded in both of the classes. In order to ground it better, and shared among the students, Cabri has been introduced by the teacher (in a deep way in class A, lightly in class B), for representing the variation of area, depending from the variation of AP.

It can be obtained with the following steps:

a) to insert the Cartesian reference system with the 'show axes"

b) to transfer (a measurement) the length of AB on axis x and the measures of perimeter and area on axis y

c) to choose the point *Area*, with coordinates the length of AP and the area of the rectangle

d) to choose the point *Perimeter*, with coordinates the length of AP and the perimeter of the rectangle

e) to use the "trace" command with points *Area* and *Perimeter*, in order to obtain the graph of function area and function perimeter, dragging point P.

The resulting figure is something like Figure 4:

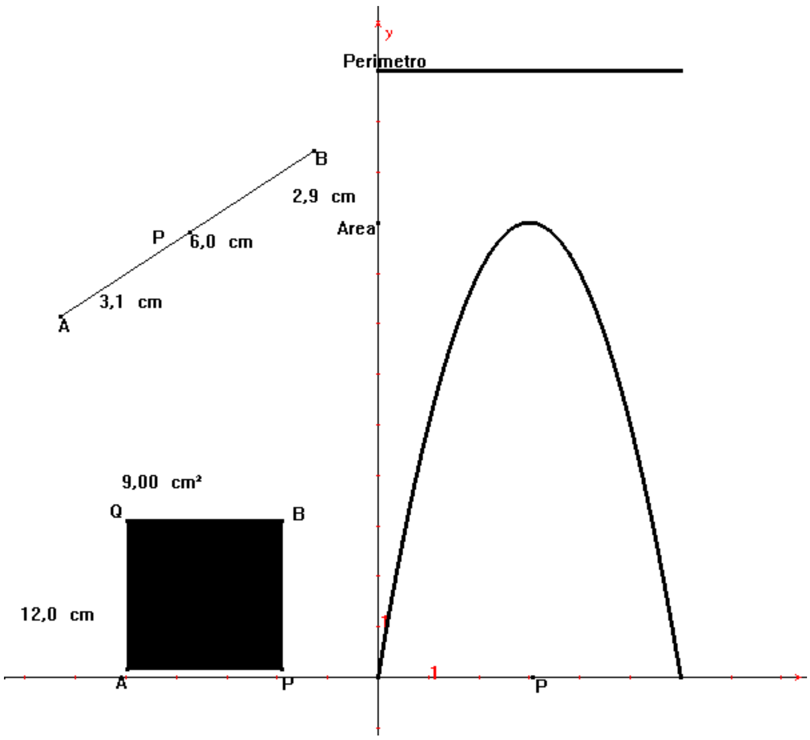

Figure 4

After an activity of this kind, all the students are absolutely convinced that the found solution is the right one. It could be interesting asking them, at this point, *why* the maximum of the area is when AP = 3, namely when the rectangle is a square. Usually the first reaction by the students is to give answers of this kind: *it is like this because it is like this*.

The more the teacher is questioning, the more the students become disoriented and impatient, because they are not able to understand what is interesting to be questioned in a problem that does not seem to reveal something new. If a balanced atmosphere between disorientation and curiosity is created, the teacher can enter the scene with a very important and significant intervention about the role of justification in Mathematics. This could be a perfect occasion to introduce students to theoretical thinking, to the role and the specificity of a theory as environment in which it is possible to *explain why*: “It is shown how the empirical approach to geometry (which is often associated with computer based dynamic geometry software) can be used as a basis for creating didactic situations in which students require proofs" (Dreyfus & Hadas, 1996).

In other words, it is a good occasion to start with a work on the role and the functions of proof in a mathematical theory: not *to convince* of the validity of a conjecture, but to precise the dependence of a conjecture from a set of propositions as basis of a theory, through the relation of logic consequence (Paola, 2004).

In class B a pair of students, after many questions from the teacher about *why* the solution works, answered: "Look Mr., if I am in the middle point and I move a bit on the left and a bit on the right, the area diminishes …"

This answer gives the teacher the possibility to act in a sort of zone of proximal development with this answer: "How can translate in symbols and in operative terms a bit on the left and a bit on the right of the middle point? $3 - x$ e $3 + x$. So, the area of the rectangle is $(3 - x)\cdot(3+ x)$, namely $9 - x^2$". At this point the students have no difficulties in recognising that the maximum is when $x = 0$. The teacher has to use this occasion for trying to explain why this answer can be considered a *proof* and so an *explanation of why* the square is the rectangle of the maximum value for the area of a set of rectangles with the same perimeter (Paola, 2004).

Obviously, the meaning of the term *why* has to be negotiated and not imposed with authority: it represents a crucial point in mathematical thinking that has to be treated more times. We may not pretend that all the students can appreciate such difficult motivations at the first occasion, because they need to share the mathematical building from a theoretical point of view.

Conclusions

The activity discussed here, clearly shows that Cabri is not only a Dynamic Geometry Software, but, more generally, a microworld (Balacheff, & Sutherland, 1994; Laborde & Strässer, 1990) that let possible the construction of mathematical teaching-learning environments. Particularly for what concerns the study of quantity variation, with Cabri it is possible to work simultaneously:

a) on the perceptive-motor aspects (e.g. the rectangle that changes its shape under the dragging of point P);

b) on the numerical aspects (e.g. the variation of the numbers representing the measures of the area, which change with different velocity, according to the respective distance from the maximum);

c) on the graphs, which favour seeing a relation between the motor-perceptive and the numerical aspects, and represent a mediation between the two, not immediately simple to see;

d) on symbolic formulas, intended also as a tool for introducing students to theoretical thinking and proof

e) on symbol role in the teaching-learning process, particularly in the construction of mathematical meanings, and more generally, in Mathematics as a subject-matter.

From the didactic point of view activities like the one presented here unveil interesting perspectives for the introduction of significant Calculus concepts. Particularly, as teachers we have to observe the awareness that students learn to have, in solving activities related to:

a) quantity change and rate of change;

b) relations between numerical, graphical and symbolic aspects involved in an activity.

Those aspects are at the core of Calculus as a subject matter, and it is possible to ground a *sensible* teaching practice on such aspects, as Galileo intended: “linked to senses and to reason”.

The same aspects in meaningful activities have the important aim to introduce students not only to some Calculus concepts, but also to them as part of a theory, particularly to theoretical knowledge and its educational function: to explain *why* a statement is true. The explanation of *why* is an essential feature of a theory, and the relative answers gain a full meaning in the theory itself.

From the point of view of research in Mathematics Education it seems particularly significant highlight that the three elements of the theoretical framework we used here in an integrated way have been useful in various ways:

- first of all, to project a teaching sequence coherent with the aim of introducing functions and topics of Calculus (particularly referring to the first element of our framework, namely the studies about DGS);

- then to observe, interpret and analyse students' behaviours and their interaction with the teacher, especially what metaphors are used, and also other non linguistic aspects, as gestures, which we are studying, even if not yet reporting here (and the second element of our framework is useful for that);
- finally, the third element (namely the instrumental approach) of the theoretical framework has revealed useful in observing and interpreting the role played by the artefact (particularly the instrumental genesis helped us in explaining the initial different students' behaviours (of class A and B) coping the same problem.

In our opinion there are strict connections between the didactic and research point of view, e.g. some studies show that providing students with problems in the form "prove that…" may prevent them from being able to attempt proving. On the contrary, providing students with problems which require and support the production of conjectures may help them in the proving phase: the hypothesis is that a cognitive continuity in the transition from exploration to proving might be constructed on the basis of the production of conjectures (Boero et al., 1996). Another example could be the important synergy existing among the kind of use of the artefact and the institutional knowledge (in the sense of Chevallard, 1992). To explain better, if a teacher thinks that the role of a proof is to convince someone that a statement is true, then the use of DGS could have opposite effect on the construction of a theory (in the sense described before, namely to explain *why*). In fact, the results of the explorations made in the DGS could convince very well, so they could render a proof (aimed at convince) complete usefulness, unless using a DGS exclusively for activities of the kind suggested in a recent article (Hadas & al., 2000).

Otherwise, if a teacher thinks that the main role of a proof is to explain *why* a statement is true, namely after being convinced, to explain the link of logic consequence between the fact observed and the previous statements, then a DGS becomes an excellent and helpful artefact for proving, just because it let possible convincing!

The problematic matter of instrumental genesis and of its difficult realisation suggests also that some schemes of use of the artefact (e.g. dragging) should be taught themselves: anyway, this is not expected at all. The difficulty showed by the students of class A (not expertise in the use of Cabri), in “seeing” what they are looking at and in using dragging in a proper way, suggests that the students must actually learn the process of exploration allowed by the instrument.

The theoretical elements used in this article are fundamental because even in teaching practice, as in Mathematics, without precise theoretical frames, sometimes one is not able to “see” what he is looking at.

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