The paper uses a semiotic lens to interpret the interactions between teacher and students, who work in small collaborative groups. This allows focusing some important strategies, called semiotic games, used by the teacher to support students mathematics learning. The semiotic games are discussed within a Vygotskian frame.

INTRODUCTION

The role of the teacher in promoting learning processes is crucial and has been analysed according to different frameworks. For example the Theory of Didactic Situations, originated by G. Brousseau (1997), defines the teacher as a didactical engineer. (S)he designs the situations and organises the milieu according to the piece of mathematics to be taught and to the features of the students (mesogenesis: see Sensevy et al., 2005); s(he) divides the activity between the teacher and the students, according to their potentialities (topogenesis); moreover the classroom interactions are pictured according to the didactic contract, that is the system of reciprocal explicit and implicit expectations between the teacher and the students as regards mathematical knowledge. Brown and McIntyre (1993) underline that the teacher works with students in classrooms and designs activity for classrooms using her/his craft knowledge, namely a knowledge largely rooted in the practice of teaching. Voigt (1995) and Lerman (1998) point out “the need for a deeper understanding of the ways in which teachers contribute to the shaping of classroom cultures” (quoted in Siemon & al., 2004). Other researchers, who work according to Vygotsky’s conceptualization of ZPD (Vygotsky, 1978, p. 84), underline that teaching consists in a process of enabling students’ potential achievements. The teacher must provide the suitable pedagogical mediation for students’ appropriation of scientific concepts (Schmittau, 2003). Within such an approach, some researchers (e.g. Bartolini & Mariotti, to appear) picture the teacher as a semiotic mediator, who promotes the evolution of signs in the classroom from the personal senses that the students give to them towards the scientific shared sense. In this case teaching is generally conceived as a system of actions that promote suitable processes of internalisation.

Our approach is in the Vygotskian stream: the teacher is seen as a semiotic mediator, who promotes students’ internalisation processes through signs. But some changes are proposed with respect to the classical Vygotskian approach. First, we extend the notion of sign to all semiotic resources used in the teaching activities: words (in oral or in written form); extra-linguistic modes of expression (gestures, glances, …); different types of inscriptions (drawings, sketches, graphs, …); different instruments (from the pencil to the most sophisticated ICT devices). Second, we consider the embodied and multimodal ways in which such resources are produced, developed and used. Within such a framework, we utilise a wider semiotic lens (the semiotic bundle,
sketched below) to focus the interactions between teacher and students. Our semiotic lens allows framing and describing an important semiotic phenomenon, which we have called *semiotic games*. The semiotic games practise is rooted in the craft knowledge of the teacher, and most of times is pursued unconsicously by her/him. Once explicit, it can be used to properly design the teacher’s intervention strategies in the classroom for supporting students’ internalisation processes.

In the following three sections we discuss: (i) The multimodal paradigm and the semiotic tools suitable for describing mathematics learning processes. (ii) An emblematic example, through which the notion of semiotic games is introduced (the main result of the paper). (iii) Some didactical consequences.

**FROM THE MULTIMODALITY OF LEARNING PROCESSES TO THE SEMIOTIC BUNDLE**

The notion of *multimodality* has evolved within the paradigm of *embodiment*, which has been developed in these last years (Wilson, 2002). Embodiment is a movement afoot in cognitive science that grants the body a central role in shaping the mind. It concerns different disciplines, e.g. cognitive science and neuroscience, interested with how the body is involved in thinking and learning. The new stance emphasizes sensory and motor functions, as well as their importance for successful interaction with the environment. A major consequence is that the boundaries among perception, action and cognition become *porous* (Seitz, 2000). Concepts are so analysed not on the basis of “formal abstract models, totally unrelated to the life of the body, and of the brain regions governing the body’s functioning in the world” (Gallese & Lakoff, 2005, p.455), but considering the *multimodality* of our cognitive performances. Verbal language itself (e.g. metaphorical productions) is part of these cognitive multimodal activities (*ibid.*).

In general, semiotics is a powerful tool for observing the didactical processes. However, the classical semiotic approaches put strong limitations upon the structure of the semiotic systems they consider. These result too narrow for interpreting the didactical phenomena in the classroom. This happens for two reasons:

(i) Students and teachers use a variety of semiotic resources in the classroom: gestures, glances, drawings and extra-linguistic modes of expression. But some of them do not satisfy the requirements of the classical definitions for semiotic systems as discussed in the literature (e.g. in Duval, 2006 or in Arzarello, 2006b).

(ii) The way in which such different resources are activated is multimodal. It is necessary to carefully study the relationships within and among those, which are active at the same moment, and their dynamic developing in time.

Hence we need a broader theoretical tool for analysing the semiotic resources in the classroom. This tool is the *Semiotic Bundle*, introduced in Arzarello (2006b). It encompasses all the classical semiotic systems as particular cases. Hence, it is coherent with the classical semiotic analysis, but broaden it and allows getting new results and framing the old ones within a unitary wider picture. Roughly speaking
(for a full description see Arzarello, 2006b), while the classical semiotic systems concern very structured systems, whose rules of sign production and manipulation are very precise algorithms (from the oral and written language to the algebraic or Cartesian register) the semiotic bundle includes all signs produced by actions that have an intentional character (e.g. speaking, writing, drawing, gesticulating, handling an artefact, etc.) and whose modes of production and transformation (e.g. for gesturing or drawing) may encompass also approaches less deterministic and more idiosyncratic than algorithms. A semiotic bundle is a dynamic structure, where such different resources coexist and develop with their mutual relationships, according to the multimodal paradigm. Hence it allows considering a variety of resources, which span from the compositional systems, usually studied in traditional semiotics (e.g. formal languages) to the open sets of signs (e.g. sketches, drawings, gestures). An example of semiotic bundle is represented by the unity speech-gesture. It has been written that “gesture and language are one system” (McNeill, 1992, p.2): from our point of view, gesture and language are two components of the same semiotic bundle. Research on gestures has already shown important relationships between them (e.g. match Vs. mismatch, see Goldin-Meadow, 2003).

We have used the semiotic bundle to analyse different classroom stories (Arzarello et al., 2006a; Arzarello, 2006b). It has revealed particularly useful for studying several didactic phenomena that happen in the classroom, especially some interactions between teacher and students, who work in small groups. We have called them the semiotic games. They consist in strategies of intervention in the classroom that many times the teachers activate unconsciously; once they become aware of them, they can use them in a more scientific way for improving their students achievements. We shall introduce the semiotic games through an emblematic example in the following section; further discussion is found in Sabena (2007). We have observed such games in different classes and with students of different ages (from elementary to secondary school).

THE SEMIOTIC GAMES THROUGH AN EMBLEMATIC EXAMPLE

The activity we shall comment concerns students attending the third year of secondary school (11th grade; 16-17 years old). They attend a scientific course with 5 classes of mathematics per week, including the use of computers with mathematical software. These students are early introduced to the fundamental concepts of Calculus since the beginning of high school (9th grade); they have the habit of using different types of software (Excel, Derive, Cabri_Géomètre, TI-Interactive, Graphic Calculus: see Arzarello et al. 2006a) to represent functions, both using their Cartesian graphs and their algebraic representations. Students are familiar with problem solving activities, as well as with interactions in small groups. The methodology of mathematical discussion is aimed at favouring the social interaction and the construction of a shared knowledge.

We will comment some excerpts from the activity of a group of three students: C, G, S. They are clever pupils, who participate to classroom activities with interest and
active involvement. In the episodes we present, there is also the teacher (T), whose role is crucial and will be carefully analysed: he is not always with these students, but passes from one group to the other (the class has been divided into 6 small groups of 3-4 students each). The excerpts illustrate what is happening after the group has done some exploring activities on one PC, where Graphic Calculus produces the graphs of Figure 1. Their task is to explain the reasons why the slope of the ‘quasi-tangent’ (see the box with Fig. 1) is changing in that way. The students know the concepts of increasing/decreasing functions but they do not yet know the formal notion of derivatives. Moreover they are able in using Graphic Calculus and know that the ‘quasi-tangent’ is not the real tangent, because of discrete approximations.

Typically their first explanations are confuse (see Episode A) and expressed in a semiotic bundle, where the speech is not the fundamental part. In fact, the main component of the semiotic bundle consists in the multimodal use of different resources, especially gestures, to figure out what happens on the screen. Figures 2 show how C captures and embodies the inscriptions in the screen through his gestures. More precisely, the evolution of the gesture from Fig. 2a to Fig. 2e illustrates a sprouting concept not completely formulated in words. It could be phrased so: "the quasi-tangent is joining pairs of points whose x-coordinates are equidistant, but it is not the same for the corresponding y-coordinates: the farther they are the steepest is the quasi-tangent". These ideas are jointly expressed through gestures and words. In fact, C’s words refer only to the ‘quasi-tangent’ line and express the fact that the interval $\Delta x$ is always the same; the remaining part is expressed through gestures. Only later the concept will be expressed verbally. We call the gesture in Fig. 2b the basic sign (the thumb and the index getting near each other): in fact it triggers a semiotic genesis of signs within the semiotic bundle that is being shared among the students (and the teacher, as we shall see).

In episode A the student C is interacting with the teacher, whose role will be analysed later. At the moment we limit ourselves to focus on the specific content of his interventions in this episode. In #1 he is echoing C’s words (#0) using a more technical word ($\delta x$), namely he gives the scientific name to the concept expressed by C and C shows that he understands what the teacher is saying (#2). C’s
attention is concentrated on the relationships between the $\Delta x$ and the corresponding $\Delta y$ variations. Gesture and speech both contribute to express the covariation between $\Delta x$ and $\Delta y$, underlining the case when the variations of $\Delta y$ become bigger corresponding to fixed values of $\Delta x$. Figures 2 and the corresponding speech illustrate the multimodality of C’s actions: the student is speaking and simultaneously gesturing. C grasps the relationship of covariance with some difficulty, as the misunderstanding in sentences from #5 to #9 shows. Here the intervention of the Teacher (#6) supports C in the stream of his reasoning, which can continue and culminates in sentence #11, where the gesture (see the overturned arm in Fig. 2e) gives evidence that C realised that the covariance concerns also negative slopes (but he does not use such words).

**Episode A.** (duration: 9 seconds; about one hour after the beginning of the activity).

0 C: The X-interval is the same …

1 T: The X-interval is the same; delta-x [$\Delta x$] is fixed.

2 C: Delta…eh, indeed, however…however there are some points where… to explain it … one can say that this straight line must join two points on the Y axis, which are farther each other. (Figs. 2a, 2b)

3 C: Hence it is steeper towards...(Fig. 2c)

4 G: Yes!

5 C: Let us say towards this side. When, here, …when …however it must join two points, which are farther, that is there is less…less distance. (Fig. 2d)

6 T: More or less far?

7 C: Less…less far [he corrects what he said in #5].

8 T: Ehh?

9 C: On the Y axis I am saying!

10 T: Yes!
C: It slants softly from this side (Fig. 2e).

After a few seconds there is an important interaction among the teacher and the students (Episode B), which is emblematic of the strategy used by the teacher to work with students for promoting and facilitating their mathematics learning.

**Episode B.** (duration: 34 seconds; a few seconds after Episode A).

18 T: Hence let us say, in this moment if I understood properly, with a fixed delta-x, which is a constant,… (Fig. 2f).

19 C: Yes!

20 S: Yes!

21 T: It… is joining some points with delta-y, which are near (Fig. 2f).

22 C: In fact, now they [the points on the graph] are more and more…

23 T: It is decreasing, is it so? [with reference to $\Delta y$]

24 S: Yes!

25 C: …they [their ordinates] are less and less far. In fact, the slope... I do not know how to say it,……the slope is going towards zero degrees.

26 T: Uh, uh.

27 C: Let us say so…

28 S: Ok, at a certain point here delta-y over delta-x reaches here…

29 C: …the points are less and less far.

30 T: Sure!

31 S: …a point, which is zero.

[sentences of C(#27, #29), of S(#28, #31) and of T (#30) are intertwined each other]

This episode shows an important aspect of the teacher’s role: his interventions are crucial to foster the positive development of the situation. This appears both in his gestures and in his speech. In fact he summarises the fundamental facts that the students have already pointed out: the covariance between $\Delta x$ and $\Delta y$ and the trend of this relationship nearby the stationary point (we skipped this part). To do that he exploits the expressive power of the semiotic bundle used by C and S. In fact he uses twice the basic sign: in #18 to underline the fixed $\Delta x$ and in #21 to refer to the corresponding $\Delta y$ and to its smallness nearby the local maximum $x$ (non redundant gestures: see Kita, 2000). In the second part of the episode (from #22 on) we see the immediate consequence of the strategy used by the teacher. C has understood the relationship between the covariance and the phenomena seen on the screen nearby the stationary point. But once more he is (#25) unable to express the concept through speech. On the contrary, S uses the words previously introduced by the teacher (#18, #21) and converts what C was expressing in a multimodal way through gestures and (metaphoric) speech into a fresh semiotic register. His words in fact are an oral form of the symbolic language of mathematics: the semiotic bundle now contains the official language of Calculus. His sentences #28, #31 represent this formula. The episode illustrates what we call semiotic games of the teacher. Typically, the teacher uses the multimodality of the semiotic bundle produced by the students to develop his semiotic mediation. Let us consider #18 and #21 and Fig. 2f. The teacher mimics one of the signs produced in that moment by the students (the basic sign) but
simultaneously he uses different words: precisely, while the students use an imprecise verbal explanation of the mathematical situation, he introduces precise words to describe it (#18, #21, #23) or to confirm the words of S (#30). Namely, the teacher uses one of the shared resources (gestures) to enter in a consonant communicative attitude with his students and another one (speech) to push them towards the scientific meaning of what they are considering. This strategy is developed when the non verbal resources utilised by the students reveal to the teacher that they are in ZPD. Typically, the students explain a new mathematical situation producing simultaneously gestures and speech (or other signs) within a semiotic bundle: their explanation through gestures seems promising but their words are very imprecise or wrong and the teacher mimics the former but pushes the latter towards the right form.

CONCLUSIONS

Semiotic games are typical communication strategies among subjects, who share the same semiotic resources in a specific situation. The teacher uses the semiotic bundle both as a tool to diagnose the ZPD of his students and as a shared store of semiotic resources. Through them he can develop his semiotic mediation, which pushes their knowledge towards the scientific one. Roughly speaking, semiotic games seem good for focussing further how “the signs act as an instrument of psychological activity in a manner analogous to the role of a tool in labour” (Vygotsky, 1978, p. 52) and how the teacher can promote their production and internalisation. The space does not allow to give more details (they are in Arzarello & Robutti, to appear) and we limit ourselves to sketchily draw some didactical consequences. A first point is that students are exposed in classrooms to cultural and institutional signs that they do not control so much. A second point is that learning consists in students’ personal appropriation of the signs meaning, fostered by strong social interactions, under the coaching of the teacher. As a consequence, their gestures within the semiotic bundle (included their relationships with the other signs alive in the bundle) become a powerful mediating tool between signs and thought. From a functional point of view, gestures can act as “personal signs”; while the semiotic game of the teacher starts from them to support the transition to their scientific meaning. Semiotic games constitute an important step in the process of appropriation of the culturally shared meaning of signs, that is they are an important step in learning. They give the students the opportunity of entering in resonance with teacher’s language and through it with the institutional knowledge. However, in order that such opportunities can be concretely accomplished, the teacher must be aware of the role that multimodality and semiotic games can play in communicating and in productive thinking. Awareness is necessary for reproducing the conditions that foster positive didactic experiences and for adapting the intervention techniques to the specific didactic activity. E.g. in this report we have considered teacher’s interventions in small collaborative groups. In a whole class discussion, the typology of semiotic games to develop will change, depending on the relationships within and among the different components of the semiotic bundles produced and shared in the classroom. This issue
suggests new researches on the role of the teacher in the classroom, where the semiotic lens can again constitute a crucial investigating tool.

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References