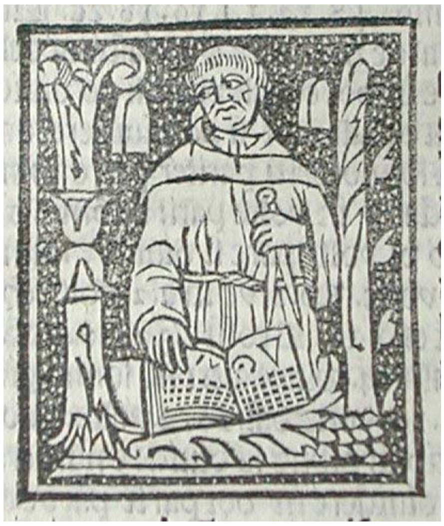
HISTORY AS A CROSSROADS OF MATHEMATICAL CULTURE AND EDUCATIONAL NEEDS IN THE CLASSROOM

by Fulvia Furinghetti and Domingo Paola



INTRODUCTION

In this article we discuss how the history of mathematics may be used as a support in mathematics teaching. We face the question from the point of view of teachers. Let us suppose that a teacher is charmed by history (because he/she likes history, because he/she trusts history as a support for teaching, because a colleague he/she admires uses history...). This teacher has to design the instructional sequence he/she wishes to implement in classroom on the ground of the two domains schematised in Fig.1. This is also what is done by curriculum developers and by researchers in mathematics education when designing an instructional sequence. Behind Fig.1 there are the assumptions that the two domains of history and of mathematics teaching are overlapping and that the potentialities/facilities offered by the overlapping part in Fig.1 have to be conveniently exploited.

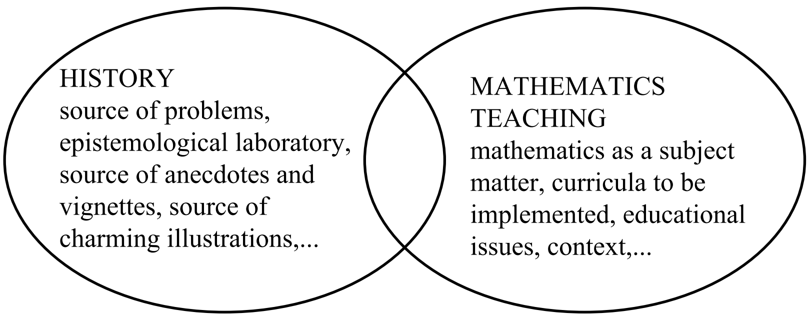


Fig.1. Domains involved in the design of an instructional sequence including the history of mathematics

From the literature in this field, we have singled out two possible approaches in the use of the history of mathematics in the classroom, see (Furinghetti, 1997). In the first approach, history has the main function of promoting students' interest in mathematics. In the second approach, history is integrated into mathematics teaching as an aid to pursuing mathematical goals from a new point of view and in a new work environment. According to some authors (Umberto Eco, Hayden White) historiography (in general, not only of mathematics) may be seen as an artifact. As an artifact, the history of mathematics, together with its modes of use as interpreted by the user (teacher or student), may become a teaching/learning tool (if the user is able to uses it for his/her purposes).

When history is used to pursue specific mathematics goals (second approach), designing an instructional sequence requires competent work in the field of history and of education. In Fig.2, we give a schema of the path for preparing historical materials for the classroom. We are aware that a teacher left alone may find it difficult to carry out this path because the many issues involved. Very often in this case a manual of history of mathematics may be not enough and the teacher has to go to primary sources. The schema emphasizes the wide range of competencies required by the teacher and thus reminds us once again of the importance of the perennial problem of teacher education in the history of mathematics, see (Schubring et al., 2000).

Of course, many problems are bypassed if teachers are provided with a ready-made instructional sequence to be implemented in classroom. But the best way would be to create environments in which teachers have the opportunity to make their choices within a range of possibilities offered by experts and conveniently presented to them. In this way, teachers should be *actively* involved in all phases of the work (designing, implementation).

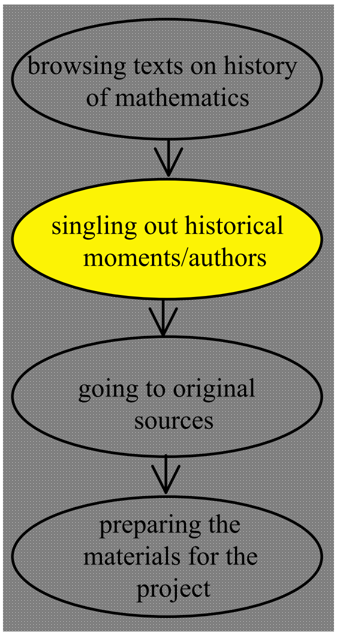


Fig.2. The path to preparing historical materials for the classroom

On these premises, we have designed a hypertext for teachers which contains the historical and educational materials suitable for designing an instructional sequence. In this paper, we describe the main lines of our project, which is inspired by our trust in the efficacy of the use of history in mathematics teaching and by the need to express an explicit solidarity with the teachers wishing to try this in their classrooms. Our aim is to outline a model of what may be a support for teachers. A fundamental requirement of this support is to guide teachers without affecting their autonomy in teaching.

THE PROJECT

The subject treated in the hypertext is probability for secondary school level (students aged about 15). All curriculum designers agree that some elements of probability have to be taught; nevertheless, the place of this part of mathematics in the curricula of the various countries is different. In the studies of mathematics education, probability is not the most studied subject but interesting papers have been published. For example, in the last conference of the International Group for the Psychology of Mathematics Education, two out of 170 research reports, one out of 100 short oral communications and two out of 40 poster presentations concerned probability, see (Van den Heuvel-Panhuizen, 2001). On the historical side, the book *History in Mathematics Education: the ICMI Study* edited by John Fauvel and Jan Van Maanen (Kluwer, Dordrecht-Boston-London, 2000), which provides a comprehensive review of research and practice on the use of history in mathematics education, contains parts treating the history of probability in teaching. All these facts show concern for the need to introduce students to elements of probability. At the same time, they show the difficulties encompassed by such an approach. As Lakoma (2000a) put it:

> Probabilistic concepts cannot be understood in depth by simply giving their logical connections to other concepts and their place in modern probability theory, founded for instance on Kolmogorov's axioms, which are too abstract to be understood by students. Didactically, a heuristic (non-axiomatic) approach is needed, which presents stochastics as a live part of mathematics, making possible the solution of real problems by describing real situations on the basis of simple models which have great explanatory value. (p.248)

Also, Lakoma points out that probability has a dual nature: epistemological (mainly linked to our beliefs, conviction or confidence in an argument concerning a phenomenon), and aleatory (related to the physical structure of the random mechanisms under consideration). She notes that:

> Development of the dual nature of probability goes in parallel with the emergence and establishment of the concept of expectation (expected value). (p.250)

In the teaching of probability, the consequence of this dual nature is that

> [...] probability and expectation can and should be introduced and developed in parallel, while always keeping in mind that it is necessary to distinguish and contrast them; e.g. one may consider probabilistic problems in connection with answering not only the question "How often?" but also the question "Is it worthwhile?" (p.250)

The dual nature of probability and the role of expectation appear clearly in the early attempts of a solution given to problems which focus on estimation of chances for winning a game or on the distribution of a stake. This kind of problem is described in (Smith, 1959) as follows:

> Italian writers of the fifteenth and sixteenth century, notably Pacioli (1494), Tartaglia (1556), and Cardan (1545), had discussed the problem of the division of a stake between two players whose game was interrupted before its close. The problem [together with other problems of chance] was proposed to Pascal and Fermat, probably in 1654, by the Chevalier de Méré, a gambler who is said to have had unusual ability "even for the mathematics." The correspondence which ensued between Fermat and Pascal was fundamental in the development of modern concepts of probability and it is unfortunate that the introductory letter from Pascal to Fermat is no longer extant. (p.546)

The story of these first attempts is the scenario we propose to teachers who wish to introduce their students to the first elements of probability. These teachers often have only few and rough ideas about probability but, as mentioned earlier, we do not want to give them an instructional sequence already prepared by us, ready to be reproduced in classroom. We wish to make them aware of the nature of the subject and to point them in the organization of an instructional sequence. For this purpose, we have designed a hypertext in which they may find the materials suitable to organize an instructional sequence on probability based on history, i.e.

- Historical materials
- Articles describing and discussing experiments on the introduction of probability in the classroom
- Articles on the history of probability
- Original passages taken from old books
- Illustrations taken from old books (front covers, portraits, figures in the treatises)
- Passages from treatises on the history of mathematics.

In our hypertext many parts are written in Italian, since it has been conceived for Italian teachers. Nevertheless, many of the historical materials are easily found also in English and in other languages. Of course, the French teachers may read the original writings of French authors, see (IREM, 1995). Articles (both historical and educational) are mainly written in English.

HISTORICAL MATERIALS

The core of the hypertext is the following problem: "How can the stake be divided in a game if the game is interrupted before one of the two players has achieved the winning score? [It is assumed that the two players are of the same value]. " This problem is known as the "problem of points". If A and B are the two players, the data of the problem are: the number n of points required to win the game; the numbers a and b of points gained by the two players A and B respectively when the game is interrupted. We summarize these data with the notation [n: a ; b], where $a<n$ and $b<n$. In his famous treatise *Summa de arithmetica geometria proportioni et proportionalità* (part I, distinctio IX, trac. X de straordinariis, cc. 197 and verso), Luca Pacioli (Italy, about 1445-?) gave his solution

in the case [60: 50 ; 20]. He divided the stake in parts proportional to the points won by the players, i.e.

Total points : points of the player A = stake : money due to A

Many authors more or less contemporary of Pacioli's or later have dealt with problems of dividing a stake, see (Dupont, 1986; Garibaldi, 1984; Simi, 1997-98). They are not so well known as those quoted by Smith (1959), but the solutions are an interesting piece in the history of probability. Filippo Calandri (Italy, about 1467-?), an abacist author of one of the first printed books of arithmetic, solved a problem of ball games with the proportional method. In another document, we find a different way of solving the problem by Calandri. He divided the stake in a proportion inverse to the number of successes being needed to achieve the winning score. This solution is important because the author considers future aspects of the game, not just what has already happened. This means that the difficulties that the players have to face, as well as the idea of the mathematical expectation, are considered. Niccolò Tartaglia (1500-1557) in his *General trattato di numeri et misure* (Part I, book XVI, cc. 265 and verso) has criticized the method of Pacioli. He noted that if the game is interrupted when one of the two players has zero points this player will receive nothing (which is unfair). A century later, we find again solutions based on proportionality (e.g. in *Arithmetica pratica utilissima* by Francesco Pagani, printed in Ferrara, 1591). Later on, only 50 years before the birth of probability, we find the Italian author Lorenzo Florestani da Pescia (?-1623) writing a survey of the solutions to problems of dividing a stake and giving once more a solution which is wrong (*Pratica d'arithmetica e geometria*, Giorgio Varisco, Venice, 1603, first edition; we have consulted the second edition printed in Siena in 1682 by Stamparia del Pubblico). The almost 200 years of discussion about the solution indicates the struggles which precede the birth of probability. Also, they give insights into the links between mathematical and psychological issues (beliefs, attitudes, ...) in facing this problem. It is interesting to note that some old solutions of the problem of dividing a stake are still mentioned in early articles on probability written around the end of nineteenth century, see (Fenaroli et al., 1991).

All the solutions mentioned above and some papers written by historians that discuss them are reported in our hypertext. Not only do they provide evidence for the efforts of the early solvers who treated the problem but also give powerful ideas for discussing in the classroom the concepts which are behind probability.

In the hypertext, there is the original correspondence (in French) between Pascal and Fermat in which these two mathematicians gave their solution of the problem. The Italian translation, taken from the reader (Bottazzini et al., 1992), of some parts of this correspondence is also reported. The translation into English of letters written in 1654 (from *Oeuvres de Fermat*, ed. Tannery and Henry, v.II, 288-314, Gauthier-Villars, Paris, 1894) is reported in (Smith, 1959, 546-565). The correspondence between these two authors shows the nature of the problem and of its solution.

We point out that, according to our belief, teachers benefit from having a critical knowledge of the subject they teach. We have added to the historical materials some papers written by professional historians in which the history of probability is presented and the crucial points are discussed. Thus the teachers wishing to go in depth into the subject have the opportunity to study these articles and to reflect by themselves on the nature of the historical studies, as it is advocated by Jahnke (1994).

HOW TO WORK IN CLASSROOM

One of the authors (a teacher in secondary school) has experimented with an instructional sequence based on the materials we have described. In the following, we summarize the development of the experiment, for a full description see (Paola, 1998). The students involved were aged 15 years, the school was an Italian Classical Lyceum. The aim of the teacher was not to develop the complete theory of probability but simply to introduce his students to ways of thinking about probability. The classroom activity was carried out through discussion of the problem between students divided into groups. The teacher not only orchestrated the discussion but also acted as an observer and reported all that happened in the classroom. Initially, all students agreed that the best way to solve the problem would be to divide the stake in parts that were proportional to the scores earned by each player. The teacher easily refused this solution by proposing that one of the two players had a score of zero when the game was interrupted. After discussion of this particular case, another group of students proposed other ways of solving it that did not satisfy their classmates. At this point, the teacher read Pacioli's solution, which is similar to that of the students, allowing them to see that an important historical character followed the same process that they did. The students seemed ready to approach the concept of fair division of the stake. Additional classes were dedicated to discussing this concept but the students did not arrive at effective results on their own (i.e. they were not able to grasp the concept of probability).

What we have described is the first stage of the instructional sequence on probability that we have in mind. A further stage should be to discuss the correspondence between Blaise Pascal and Pierre de Fermat (1654) concerning the problem in question. In the letters of these authors, we find the solution in the case of two players under the hypothesis that at any moment of the game they have the same probability of winning a single point. The required division of the stake is done on the grounds of the probability that they have of winning when the game is interrupted, see (Mahoney, 1994; Smith, 1959). The method of solution and the calculations necessary to carry it out are explained in the treatise *Traité du triangle arithmétique* by Pascal, printed in 1654 and published in 1665. Thus we see that the instructional sequence we are discussing has further developments also outside probability. The Pascal triangles is a popular subject in teaching all over the world, see the Chinese experiment reported in (Hui-Yu Su, 2000).

NOT ONLY HISTORY

The fascinating history of the problem of dividing a stake is not enough to guarantee the success of the instructional sequence when it is implemented in classroom. The way the instructional sequence is carried out is crucial. In the hypertext, we report on some articles on mathematics education that give hints to teachers wishing to experiment with the sequence. The article (Paola, 1988), which contains a careful description of the experiment, illustrates a method of working in classroom that proved to be efficient. The goal is not that students learn the history of probability but that they *work on* probability. History offers a good context for this work because it offers nice problems and a variety of solutions.

We have seen that in the experiment described, in (Paola, 1998), the statement of the problem was presented and the students were asked to solve it. The students worked in small collaborative groups. Afterwards they put forward their solutions, which were discussed by the whole class under the control of the teacher. At the end, Pacioli's solution was presented and compared with the solutions which emerged in the classroom.

A different methodology may consist in presenting statements of the problem of the stake together with the related solutions as found in old texts and afterwards to let students reflect on them. This should make it possible to generate a discussion among students centered on the questions:

- which solutions are correct
- why a given solution is not acceptable
- which ideas are behind the wrong and the right solutions.

In (Lakoma 1999; Lakoma, 2000b) various experiments on the use of old problems are reported. For example, students worked on the Méré problem stated as follows:

> A play of two persons consists of several games, in which each player has the same chance of winning. The winner is the player who wins 6 games as the first. The play was stopped when the player A has won 4 games, and B-3 games. How to share the stake between them? (Lakoma, 2000b, p.175)

When we mention students' reflection we are thinking not of a passive reflection but of an active interpretation of the thought of the authors considered, which is carried out in small collaborative groups. The students have to put forward their ideas, agreeing or contrasting historical ideas. We intend that the comparison of the solutions of the different authors should lead to the emergence of the need to find a solution satisfying the spirit of the problem.

We see that this way of dealing with history has precise teaching implications. On the one hand, we are asking for a kind of devolution of authority from the teacher to the students so that they feel responsible for the ideas circulating in the classroom. On the other hand, we expect that the teacher directs the traffic of information circulating in classroom and is ready to answer questions as well as to grasp the good ideas emerging from the discussion. The way of using history which we advocate will not produce the expected benefits if the teacher is not adopting this method of teaching. For this reason, in our hypertext we have also put articles illustrating the educational issues which we consider important in carrying out an instructional sequence on probability: these articles deal with teacher's role, classroom discussion and collaborative groups.

CONCLUSIONS

The description of our hypertext has been an occasion to reflect on what there is behind an instructional sequence that is based on the history of mathematics. As sketched in Fig.3, we see that there are a large number of competencies required in different fields: mathematics, mathematics education, history, education and communication. The interlacement of all these factors is so strict that neglecting one of them changes the output of an instructional sequence when implemented in the classroom.

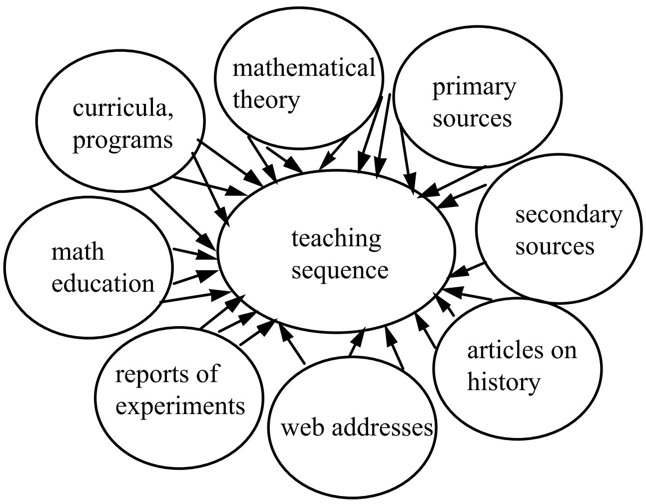


Fig.3. Behind an instructional sequence based on history

We stress that the choice itself of the historical materials to be presented in the classroom has to be made on the light of the educational factors one has in mind. In our case, the solutions worked out by the Italian authors were chosen because discussing them is within the grasp of the students. These authors deal with problems having simple statements and based on elementary mathematics. Through the ancient solutions some misconceptions held by students may be analyzed when these misconceptions are in action and show their weakness. We have also considered the fact that the words of these historical authors keep the charm of the things of the past.

Our hypertext is still in progress and we are aware that it may be improved. Nevertheless, we wish to discuss its content as an example of what has to be a valid support for teachers to encourage them to use history. Also, as we have experienced in some courses, the philosophy underlying our hypertext project may be a model for teacher training in the use of the history of mathematics (in general, not only in probability).

REFERENCES

Bottazzini, U., Freguglia, P. & Toti Rigatelli, L.: 1992, *Fonti per la storia della matematica*, Sansoni, Firenze.

Dupont, P.: 1986, 'Storia e didattica della definizione classica della probabilità', *L'educazione matematica*, a. VII, Suppl., 1-27.

Fenaroli, G., Furinghetti, F. & Somaglia, A. M.: 1991, 'Lavori relativi al calcolo delle probabilità pubblicati su periodici italiani tra la fine dell'Ottocento ed i primi del Novecento', in M. Galuzzi (editor), *Atti delle giornate di storia della matematica* (Cetraro, 1988), 105-141.

Furinghetti, F.: 1997, 'History of mathematics, mathematics education, school practice: case studies linking different domains', *For the learning of mathematics*, v.17, n.1, 55-61.

Furinghetti, F.: 2002, 'On the role of the history of mathematics in mathematics education', in I. Vakalis, D. Hughes Hall, C. Kourouniotis, D. Quinney & C. Tzanakis (editors), *Proceedings of ITCM2* (Hersonissos, Crete, Greece), CD-Rom, J. Wiley & Sons.

Garibaldi, A.C.: 1984, 'Sulla preistoria del calcolo delle probabilità', in O. Montaldo & L. Grugnetti (ediors), Atti del convegno «La storia delle matematiche in Italia» (Cagliari), 377-384.

HPM Newsletter, editor P. Ransom from number 44 (November 2000)

IREM de Basse Normandie (editor): 1995, *La création du calcul des probabilités et la loi des grands nombres de Pascal à Poisson*.

Jahnke, H.N. 1994: 'The historical dimension of mathematical understanding - Objectifying the subjective', in J.P. Ponte & J.F. Matos (editors), *Proceedings of PME XVIII* (Lisbon), v.I, 139-156.

Lakoma, E.: 1999, 'On the duality of the probability concept – from the epistemological point of view', in P. Radelet (editor), *Proceedings of the "Third European summer university in history and epistemology in mathematics education"* (Louvain-la-Neuve, Leuven), v.II, 395-409.

Lakoma, E.: 2000a, 'Stochastics teaching and cognitive development' and 'How may history help the teaching of probability concepts', in J. Fauvel, & J. Van Maanen (editors), *History in mathematics education: the ICMI Study* (Marseille, 1998), Kluwer, Dordrecht-Boston-London, 74-77 and 248-252.

Lakoma, E.: 2000b, 'On the effectiveness of history in teaching probability and statistics', in W.-S. Horng & F.-L. Lin (editors), *Proceedings of the HPM 2000 Conference History in mathematics education. Challenges for a new millennium. A satellite meeting of ICME-9*, v.I, 171-177.

Mahoney, M.S.: 1994, *The mathematical career of Pierre de Fermat*, Princeton, University Press, Princeton (402-410).

Pacioli Luca da Borgo: 1494, *Summa de arithmetica geometria proportioni et proportionalità*. We consulted the copy of the first edition, printed in 1523 (Toscolano, Benaco Lake) by Paganino de Paganinis from Brescia, who was also a printer in Venice. The figures without caption in our paper are taken from this edition.

Paola, D.: 1998, 'Il problema delle parti: Prassi didattica e storia della matematica', *Didattica delle scienze e informatica nella scuola*, a. XXXIV, n.198, 31-36.

Schubring, G. et al.: 2000, 'History of mathematics for trainee teachers', in J. Fauvel, & J. Van Maanen (editors), *History in mathematics education: the ICMI Study* (Marseille, 1998), Kluwer, Dordrecht-Boston-London, 91-142.

Simi, A.: 1997-98, 'Il problema delle parti. Alcune soluzioni anteriori alla nascita del calcolo delle probabilità', *Nuova secondaria*, XV, n.2, 34-38.

Smith, D. E.: 1959, *A source book in mathematics*, Dover, New York, v.2.

Su, H.-Y.: 2000, 'A teaching report of "Using ancient mathematical text in classroom": Pascal triangle', in W.-S. Horng & F.-L. Lin (editors), *Proceedings of the HPM 2000 Conference History in mathematics education. Challenges for a new millennium. A satellite meeting of ICME-9*, v.II, 103-112.

Van den Heuvel-Panhuizen, M. (editor): 2001, *Proceedings of 25th conference of the international Group for the Psychology in Mathematics Education* (Utrecht, The Netherlands), 4 volumes, also in CD-ROM.

The editor wishes to thank Helen Easton for her assistance with translation and interpretation.

The authors wish to thanks John Earle for editing the paper.

Keywords: Probability, historical texts, classroom practice

Authors:
Fulvia Furinghetti, Dipartimento di Matematica dell'Università di Genova (Italy).
email: furinghe@dima.unige.it
Domingo Paola, Liceo Scientifico 'A. Issel', Finale Ligure, Sv (Italy)
email: paola.domingo@mail.sirio.it